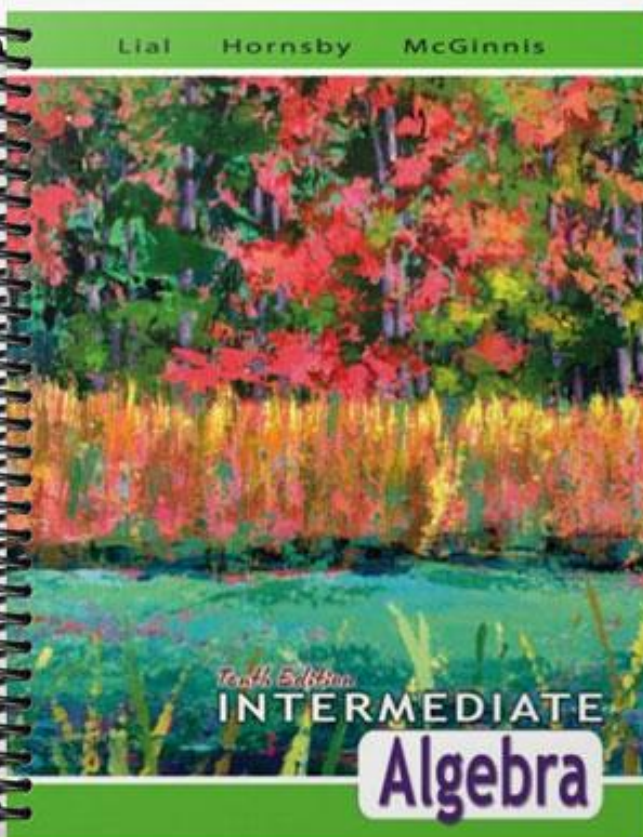


SOLUTIONS MANUAL

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Tenth Edition

INTERMEDIATE

Algebra

CHAPTER 2 LINEAR EQUATIONS, INEQUALITIES, AND APPLICATIONS

2.1 Linear Equations in One Variable

2.1 Classroom Examples

1A. (a) $9x = 10$ is an *equation* because it contains an equals sign.

(b) $9x + 10$ is an *expression* because it does not contain an equals sign.

(c) $3 + 5x - 8x + 9$ is an *expression* because it does not contain an equals sign.

(d) $3 + 5x = -8x + 9$ is an *equation* because it contains an equals sign.

1B.	$4x + 8x = -9 + 17x - 1$	<i>Original equation</i>
	$12x = 17x - 10$	<i>Combine terms.</i>
	$12x - 17x = 17x - 10 - 17x$	<i>Subtract 17x.</i>
	$-5x = -10$	<i>Combine terms.</i>
	$\frac{-5x}{-5} = \frac{-10}{-5}$	<i>Divide by -5.</i>
	$x = 2$	<i>Proposed solution</i>

Check by substituting 2 for x in the *original* equation.

$4x + 8x = -9 + 17x - 1$	<i>Original equation</i>
$4(2) + 8(2) = -9 + 17(2) - 1$	? <i>Let $x = 2$.</i>
$8 + 16 = -9 + 34 - 1$	?
$24 = 25 - 1$	?
$24 = 24$	<i>True</i>

The solution set is $\{2\}$.

2.

$$6 - (4 + m) = 8m - 2(3m + 5)$$

$$6 - 4 - m = 8m - 6m - 10$$

Distributive property

$$2 - m = 2m - 10$$

Combine terms.

$$2 - m + m + 10 = 2m - 10 + m + 10$$

Add m; add 10.

$$12 = 3m$$

Combine terms.

$$\frac{12}{3} = \frac{3m}{3}$$

Divide by 3.

$$4 = m$$

Proposed solution

We will use the following notation to indicate the value of each side of the original equation after we have substituted the proposed solution and simplified.

Check $m = 4$: $-2 = 32 - 34$ *True*

The solution set is $\{4\}$.

3. $\frac{k+1}{2} + \frac{k+3}{4} = \frac{1}{2}$

Multiply each side by the LCD, 4, and use the distributive property.

$$4\left(\frac{k+1}{2}\right) + 4\left(\frac{k+3}{4}\right) = 4\left(\frac{1}{2}\right)$$

$$2(k+1) + 1(k+3) = 2$$

$$2k + 2 + k + 3 = 2$$

$$3k + 5 = 2$$

$$3k = -3$$

Subtract 5.

$$k = -1$$

Divide by 3.

Check $k = -1$: $0 + \frac{2}{4} = \frac{1}{2}$ *True*

The solution set is $\{-1\}$.

4. $0.02(60) + 0.04p = 0.03(50 + p)$

Multiply each term by 100.

$$2(60) + 4p = 3(50 + p)$$

$$120 + 4p = 150 + 3p$$

$$4p = 30 + 3p$$

$$p = 30$$

Check $p = 30$: $1.2 + 1.2 = 2.4$ *True*

The solution set is $\{30\}$.

5. (a) $5(x+2) - 2(x+1) = 3x+1$

$$5x + 10 - 2x - 2 = 3x + 1$$

$$3x + 8 = 3x + 1$$

$$3x + 8 - 3x = 3x + 1 - 3x$$

Subtract 3x.

$$8 = 1$$

False

Since the result, $8 = 1$, is *false*, the equation has no solution and is called a *contradiction*.

The solution set is $\emptyset$.

(b) $\frac{x+1}{3} + \frac{2x}{3} = x + \frac{1}{3}$

Multiply each side by the LCD, 3, and use the distributive property.

$$3\left(\frac{x+1}{3}\right) + 3\left(\frac{2x}{3}\right) = 3\left(x + \frac{1}{3}\right)$$

$$x + 1 + 2x = 3x + 1$$

$$3x + 1 = 3x + 1$$

This is an *identity*. Any real number will make the equation true.

The solution set is $\{\text{all real numbers}\}$.

$$\begin{aligned}
 \text{(c)} \quad & 5(3x + 1) = x + 5 \\
 & 15x + 5 = x + 5 \\
 & 14x + 5 = 5 \quad \text{Subtract } x. \\
 & 14x = 0 \quad \text{Subtract 5.} \\
 & x = 0 \quad \text{Divide by 14.}
 \end{aligned}$$

This is a *conditional equation*.

Check $x = 0$: $5(1) = 0 + 5$ *True*

The solution set is $\{0\}$.

2.1 Section Exercises

- A.** $3x + x - 1 = 0$ can be written as $4x = 1$, so it is linear.

C. $6x + 2 = 9$ is in linear form.
- B.** $8 = x^2$ is not a linear equation because the variable is squared.

D. $\frac{1}{2}x - \frac{1}{x} = 0$ is not a linear equation because there is a variable in the denominator of the second term.
- $3(x + 4) = 5x$ *Original equation*
 $3(6 + 4) = 5 \cdot 6$? *Let* $x = 6$.
 $3(10) = 30$? *Add.*
 $30 = 30$ *True*
 Since a true statement is obtained, 6 is a solution.
- $5(x + 4) - 3(x + 6) = 9(x + 1)$ *Original equation*
 $5(-2 + 4) - 3(-2 + 6) = 9(-2 + 1)$? *Let* $x = -2$.
 $5(2) - 3(4) = 9(-1)$? *Add.*
 $10 - 12 = -9$? *Multiply.*
 $-2 = -9$ *False*
 Since a false statement is obtained, -2 is not a solution.
- (a)** $3x = 6$ is an *equation* because it contains an equals sign.

(b) $3x + 6$ is an *expression* because it does not contain an equals sign.

(c) $5x + 6(x - 3) = 12x + 6$ is an *equation* because it contains an equals sign.

(d) $5x + 6(x - 3) - (12x + 6)$ is an *expression* because it does not contain an equals sign.
- The final line of the check, $-11 = -11$, does not give the solution; rather, it confirms that the solution found, in this case -3 , is correct.
- The solution contains a sign error when the distributive property was applied. The left side of the second line of the solution should be

$8x - 4x + 6$. This gives us $4x + 6 = 3x + 7$ and then $x = 1$. Thus, the correct solution is 1.

$$\begin{aligned}
 8. \quad & -(2m - 4) = -1(2m - 4), \text{ so the } - \text{ sign represents } -1. \\
 & -5m - (2m - 4) + 5 = -5m - 2m + 4 + 5 \\
 & \quad \quad \quad = -7m + 9
 \end{aligned}$$

In most of the following exercises, we do not show the checks of the solutions. To be sure that your solutions are correct, check them by substituting into the original equations.

- A conditional equation has one solution, an identity has infinitely many solutions, and a contradiction has no solutions.
- By dividing each side of the equation $8x = 7x$ by x , he possibly divided each side of the equation by 0. He should have subtracted $7x$ from each side of the equation to get $x = 0$. The solution set is $\{0\}$.
- $7x + 8 = 1$
 $7x + 8 - 8 = 1 - 8$ *Subtract 8.*
 $7x = -7$
 $\frac{7x}{7} = \frac{-7}{7}$ *Divide by 7.*
 $x = -1$

We will use the following notation to indicate the value of each side of the original equation after we have substituted the proposed solution and simplified.

Check $x = -1$: $-7 + 8 = 1$ *True*

The solution set is $\{-1\}$.

$$\begin{aligned}
 12. \quad & 5x - 4 = 21 \\
 & 5x - 4 + 4 = 21 + 4 \quad \text{Add 4.} \\
 & 5x = 25 \\
 & \frac{5x}{5} = \frac{25}{5} \quad \text{Divide by 5.} \\
 & x = 5
 \end{aligned}$$

Check $x = 5$: $25 - 4 = 21$ *True*

The solution set is $\{5\}$.

$$\begin{aligned}
 13. \quad & 5x + 2 = 3x - 6 \\
 & 5x + 2 - 3x = 3x - 6 - 3x \quad \text{Subtract } 3x. \\
 & 2x + 2 = -6 \\
 & 2x + 2 - 2 = -6 - 2 \quad \text{Subtract 2.} \\
 & 2x = -8 \\
 & \frac{2x}{2} = \frac{-8}{2} \quad \text{Divide by 2.} \\
 & x = -4
 \end{aligned}$$

Check $x = -4$: $-20 + 2 = -12 - 6$ *True*

The solution set is $\{-4\}$.

$$\begin{aligned}
 14. \quad & 9p + 1 = 7p - 9 \\
 & 9p + 1 - 7p = 7p - 9 - 7p \quad \text{Subtract } 7p. \\
 & 2p + 1 = -9 \\
 & 2p + 1 - 1 = -9 - 1 \quad \text{Subtract } 1. \\
 & 2p = -10 \\
 & \frac{2p}{2} = \frac{-10}{2} \quad \text{Divide by } 2. \\
 & p = -5
 \end{aligned}$$

The solution set is $\{-5\}$.

$$\begin{aligned}
 15. \quad & 7x - 5x + 15 = x + 8 \\
 & 2x + 15 = x + 8 \quad \text{Combine terms.} \\
 & 2x = x - 7 \quad \text{Subtract } 15. \\
 & x = -7 \quad \text{Subtract } x.
 \end{aligned}$$

The solution set is $\{-7\}$.

$$\begin{aligned}
 16. \quad & 2x + 4 - x = 4x - 5 \\
 & x + 4 = 4x - 5 \quad \text{Combine terms.} \\
 & -3x + 4 = -5 \quad \text{Subtract } 4x. \\
 & -3x = -9 \quad \text{Subtract } 4. \\
 & x = 3 \quad \text{Divide by } -3.
 \end{aligned}$$

The solution set is $\{3\}$.

$$\begin{aligned}
 17. \quad & 12w + 15w - 9 + 5 = -3w + 5 - 9 \\
 & 27w - 4 = -3w - 4 \quad \text{Combine terms.} \\
 & 30w - 4 = -4 \quad \text{Add } 3w. \\
 & 30w = 0 \quad \text{Add } 4. \\
 & w = 0 \quad \text{Divide by } 30.
 \end{aligned}$$

The solution set is $\{0\}$.

$$\begin{aligned}
 18. \quad & -4t + 5t - 8 + 4 = 6t - 4 \\
 & t - 4 = 6t - 4 \quad \text{Combine terms.} \\
 & -5t - 4 = -4 \quad \text{Subtract } 6t. \\
 & -5t = 0 \quad \text{Add } 4. \\
 & t = 0 \quad \text{Divide by } -5.
 \end{aligned}$$

The solution set is $\{0\}$.

$$\begin{aligned}
 19. \quad & 3(2t - 4) = 20 - 2t \\
 & 6t - 12 = 20 - 2t \quad \text{Distributive property} \\
 & 8t - 12 = 20 \quad \text{Add } 2t. \\
 & 8t = 32 \quad \text{Add } 12. \\
 & t = 4 \quad \text{Divide by } 8.
 \end{aligned}$$

The solution set is $\{4\}$.

$$\begin{aligned}
 20. \quad & 2(3 - 2x) = x - 4 \\
 & 6 - 4x = x - 4 \quad \text{Distributive property} \\
 & 6 - 5x = -4 \quad \text{Subtract } x. \\
 & -5x = -10 \quad \text{Subtract } 6. \\
 & x = 2 \quad \text{Divide by } -5.
 \end{aligned}$$

The solution set is $\{2\}$.

$$\begin{aligned}
 21. \quad & -5(x + 1) + 3x + 2 = 6x + 4 \\
 & -5x - 5 + 3x + 2 = 6x + 4 \quad \text{Distributive property} \\
 & -2x - 3 = 6x + 4 \quad \text{Combine terms.} \\
 & -3 = 8x + 4 \quad \text{Add } 2x. \\
 & -7 = 8x \quad \text{Subtract } 4. \\
 & -\frac{7}{8} = x \quad \text{Divide by } 8.
 \end{aligned}$$

Check Substitute $-\frac{7}{8}$ for x and show that both sides equal -1.25 . The screen shows a typical check on a calculator.

$-7/8 \times 8$	-1.25
$-5(x+1) + 3x + 2$	-1.25
$6x + 4$	-1.25

The solution set is $\{-\frac{7}{8}\}$.

$$\begin{aligned}
 22. \quad & 5(x + 3) + 4x - 5 = 4 - 2x \\
 & 5x + 15 + 4x - 5 = 4 - 2x \quad \text{Distributive property} \\
 & 9x + 10 = 4 - 2x \quad \text{Combine terms.} \\
 & 11x + 10 = 4 \quad \text{Add } 2x. \\
 & 11x = -6 \quad \text{Subtract } 10. \\
 & x = -\frac{6}{11} \quad \text{Divide by } 11.
 \end{aligned}$$

The solution set is $\{-\frac{6}{11}\}$.

$$\begin{aligned}
 23. \quad & -2x + 5x - 9 = 3(x - 4) - 5 \\
 & 3x - 9 = 3x - 12 - 5 \\
 & 3x - 9 = 3x - 17 \\
 & -9 = -17 \quad \text{False}
 \end{aligned}$$

The equation is a *contradiction*.

The solution set is $\emptyset$.

$$\begin{aligned}
 24. \quad & -6x + 2x - 11 = -2(2x - 3) + 4 \\
 & -4x - 11 = -4x + 6 + 4 \\
 & -4x - 11 = -4x + 10 \\
 & -11 = 10 \quad \text{False}
 \end{aligned}$$

The equation is a *contradiction*.

The solution set is $\emptyset$.

$$\begin{aligned}
 25. \quad & 2(x + 3) = -4(x + 1) \\
 & 2x + 6 = -4x - 4 \quad \text{Remove parentheses.} \\
 & 6x + 6 = -4 \quad \text{Add } 4x. \\
 & 6x = -10 \quad \text{Subtract } 6. \\
 & x = \frac{-10}{6} = -\frac{5}{3} \quad \text{Divide by } 6.
 \end{aligned}$$

The solution set is $\{-\frac{5}{3}\}$.

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26. $4(t - 9) = 8(t + 3)$

$$4t - 36 = 8t + 24 \quad \text{Remove parentheses.}$$

$$-4t - 36 = 24 \quad \text{Subtract } 8t.$$

$$-4t = 60 \quad \text{Add } 36.$$

$$t = -15 \quad \text{Divide by } -4.$$

The solution set is $\{-15\}$.

27. $3(2w + 1) - 2(w - 2) = 5$

$$6w + 3 - 2w + 4 = 5 \quad \text{Remove parentheses.}$$

$$4w + 7 = 5 \quad \text{Combine terms.}$$

$$4w = -2 \quad \text{Subtract } 7.$$

$$w = \frac{-2}{4} = -\frac{1}{2} \quad \text{Divide by } 4.$$

The solution set is $\{-\frac{1}{2}\}$.

28. $4(x - 2) + 2(x + 3) = 6$

$$4x - 8 + 2x + 6 = 6$$

$$6x - 2 = 6$$

$$6x = 8$$

$$x = \frac{8}{6} = \frac{4}{3}$$

The solution set is $\{\frac{4}{3}\}$.

29. $2x + 3(x - 4) = 2(x - 3)$

$$2x + 3x - 12 = 2x - 6$$

$$5x - 12 = 2x - 6$$

$$3x = 6$$

$$x = \frac{6}{3} = 2$$

The solution set is $\{2\}$.

30. $6x - 3(5x + 2) = 4(1 - x)$

$$6x - 15x - 6 = 4 - 4x$$

$$-9x - 6 = 4 - 4x$$

$$-5x = 10$$

$$x = \frac{10}{-5} = -2$$

The solution set is $\{-2\}$.

31. $6p - 4(3 - 2p) = 5(p - 4) - 10$

$$6p - 12 + 8p = 5p - 20 - 10$$

$$14p - 12 = 5p - 30$$

$$9p = -18$$

$$p = -2$$

The solution set is $\{-2\}$.

32. $-2k - 3(4 - 2k) = 2(k - 3) + 2$

$$-2k - 12 + 6k = 2k - 6 + 2$$

$$4k - 12 = 2k - 4$$

$$2k = 8$$

$$k = 4$$

The solution set is $\{4\}$.

33. $-2(t + 3) - t - 4 = -3(t + 4) + 2$

$$-2t - 6 - t - 4 = -3t - 12 + 2$$

$$-3t - 10 = -3t - 10$$

The equation is an *identity*.

The solution set is $\{\text{all real numbers}\}$.

34. $4(2d + 7) = 2d + 25 + 3(2d + 1)$

$$8d + 28 = 2d + 25 + 6d + 3$$

$$8d + 28 = 8d + 28$$

The equation is an *identity*.

The solution set is $\{\text{all real numbers}\}$.

35. $2[w - (2w + 4) + 3] = 2(w + 1)$

$$2[w - 2w - 4 + 3] = 2(w + 1)$$

$$2[-w - 1] = 2(w + 1)$$

$$-w - 1 = w + 1 \quad \text{Divide by } 2.$$

$$-1 = 2w + 1 \quad \text{Add } w.$$

$$-2 = 2w \quad \text{Subtract } 1.$$

$$-1 = w \quad \text{Divide by } 2.$$

The solution set is $\{-1\}$.

36. $4[2t - (3 - t) + 5] = -(2 + 7t)$

$$4[2t - 3 + t + 5] = -(2 + 7t)$$

$$4[3t + 2] = -(2 + 7t)$$

$$12t + 8 = -2 - 7t$$

$$19t + 8 = -2 \quad \text{Add } 7t.$$

$$19t = -10 \quad \text{Subtract } 8.$$

$$t = -\frac{10}{19} \quad \text{Divide by } 19.$$

The solution set is $\{-\frac{10}{19}\}$.

37. $-[2z - (5z + 2)] = 2 + (2z + 7)$

$$-[2z - 5z - 2] = 2 + 2z + 7$$

$$-[-3z - 2] = 2 + 2z + 7$$

$$3z + 2 = 2z + 9$$

$$z = 7$$

The solution set is $\{7\}$.

38. $-[6x - (4x + 8)] = 9 + (6x + 3)$

$$-[6x - 4x - 8] = 9 + 6x + 3$$

$$-(2x - 8) = 6x + 12$$

$$-2x + 8 = 6x + 12$$

$$-8x = 4$$

$$x = \frac{4}{-8} = -\frac{1}{2}$$

The solution set is $\{-\frac{1}{2}\}$.

39. $-3m + 6 - 5(m - 1) = -5m - (2m - 4) + 5$

$$-3m + 6 - 5m + 5 = -5m - 2m + 4 + 5$$

$$-8m + 11 = -7m + 9$$

$$-m = -2$$

$$m = 2$$

The solution set is $\{2\}$.

$$\begin{aligned}
 40. \quad & 4(k+2) - 8k - 5 = -3k + 9 - 2(k+6) \\
 & 4k + 8 - 8k - 5 = -3k + 9 - 2k - 12 \\
 & -4k + 3 = -5k - 3 \\
 & k = -6
 \end{aligned}$$

The solution set is $\{-6\}$.

$$\begin{aligned}
 41. \quad & 7[2 - (3 + 4x)] - 2x = -9 + 2(1 - 15x) \\
 & 7[2 - 3 - 4x] - 2x = -9 + 2 - 30x \\
 & 7[-1 - 4x] - 2x = -7 - 30x \\
 & -7 - 28x - 2x = -7 - 30x \\
 & -7 - 30x = -7 - 30x
 \end{aligned}$$

The equation is an *identity*.

The solution set is $\{\text{all real numbers}\}$.

$$\begin{aligned}
 42. \quad & 4[6 - (1 + 2x)] + 10x = 2(10 - 3x) + 8x \\
 & 4[6 - 1 - 2x] + 10x = 20 - 6x + 8x \\
 & 4(5 - 2x) + 10x = 20 + 2x \\
 & 20 - 8x + 10x = 20 + 2x \\
 & 20 + 2x = 20 + 2x
 \end{aligned}$$

The equation is an *identity*.

The solution set is $\{\text{all real numbers}\}$.

$$\begin{aligned}
 43. \quad & -[3x - (2x + 5)] = -4 - [3(2x - 4) - 3x] \\
 & -[3x - 2x - 5] = -4 - [6x - 12 - 3x] \\
 & -[x - 5] = -4 - [3x - 12] \\
 & -x + 5 = -4 - 3x + 12 \\
 & -x + 5 = -3x + 8 \\
 & 2x = 3 \\
 & x = \frac{3}{2}
 \end{aligned}$$

The solution set is $\{\frac{3}{2}\}$.

$$\begin{aligned}
 44. \quad & 2[-(x - 1) + 4] = 5 + [-(6x - 7) + 9x] \\
 & 2[-x + 1 + 4] = 5 + [-6x + 7 + 9x] \\
 & 2[-x + 5] = 5 + [3x + 7] \\
 & -2x + 10 = 3x + 12 \\
 & -5x = 2 \\
 & x = -\frac{2}{5}
 \end{aligned}$$

The solution set is $\{-\frac{2}{5}\}$.

45. The denominators of the fractions are 3, 4, and 1. The least common denominator is $(3)(4)(1) = 12$, since it is the smallest number into which each denominator can divide without a remainder.

46. Yes, the coefficients will be larger, but you will get the correct solution. As long as you multiply both sides of the equation by the *same* nonzero number, the resulting equation is equivalent and the solution does not change.

47. (a) We need to make the coefficient of the first term on the left an integer. Since $0.05 = \frac{5}{100}$, we multiply by 10^2 or 100. This will also take care of the second term.

(b) We need to make 0.006, 0.007, and 0.009 integers. These numbers can be written as $\frac{6}{1000}$, $\frac{7}{1000}$, and $\frac{9}{1000}$. Multiplying by 10^3 or 1000 will eliminate the decimal points (the denominators) so that all the coefficients are integers.

$$\begin{aligned}
 48. \quad & 0.06(10 - x)(100) \\
 & = 0.06(100)(10 - x) \\
 & = 6(10 - x) \\
 & = 60 - 6x \quad \text{Choice B is correct.}
 \end{aligned}$$

$$\begin{aligned}
 49. \quad & -\frac{5}{9}k = 2 \\
 & -5k = 18 \quad \text{Multiply by 9.} \\
 & k = \frac{18}{-5} = -\frac{18}{5} \quad \text{Divide by } -5.
 \end{aligned}$$

The solution set is $\{-\frac{18}{5}\}$.

$$\begin{aligned}
 50. \quad & \frac{3}{11}z = -5 \\
 & 3z = -55 \quad \text{Multiply by 11.} \\
 & z = \frac{-55}{3} = -\frac{55}{3} \quad \text{Divide by 3.}
 \end{aligned}$$

The solution set is $\{-\frac{55}{3}\}$.

$$\begin{aligned}
 51. \quad & \frac{6}{5}x = -1 \\
 & 6x = -5 \quad \text{Multiply by 5.} \\
 & x = \frac{-5}{6} = -\frac{5}{6} \quad \text{Divide by 6.}
 \end{aligned}$$

The solution set is $\{-\frac{5}{6}\}$.

$$\begin{aligned}
 52. \quad & -\frac{7}{8}r = 6 \\
 & -7r = 48 \quad \text{Multiply by 8.} \\
 & r = \frac{48}{-7} = -\frac{48}{7} \quad \text{Divide by } -7.
 \end{aligned}$$

The solution set is $\{-\frac{48}{7}\}$.

$$\begin{aligned}
 53. \quad & \frac{m}{2} + \frac{m}{3} = 5 \\
 & \quad \text{Multiply by the LCD, 6.}
 \end{aligned}$$

$$\begin{aligned}
 & 6\left(\frac{m}{2} + \frac{m}{3}\right) = 6(5) \\
 & 6\left(\frac{m}{2}\right) + 6\left(\frac{m}{3}\right) = 30 \quad \text{Distributive property} \\
 & 3m + 2m = 30 \\
 & 5m = 30 \quad \text{Add.} \\
 & m = 6 \quad \text{Divide by 5.}
 \end{aligned}$$

Check $m = 6$: $3 + 2 = 5$ True

The solution set is $\{6\}$.

54. $\frac{x}{5} - \frac{x}{4} = 1$
Multiply by the LCD, 20.

$$20\left(\frac{x}{5} - \frac{x}{4}\right) = 20(1)$$

$$20\left(\frac{x}{5}\right) - 20\left(\frac{x}{4}\right) = 20 \quad \text{Distributive property}$$

$$4x - 5x = 20$$

$$-x = 20 \quad \text{Subtract.}$$

$$x = -20 \quad \text{Multiply by } -1.$$

Check $x = -20$: $-4 + 5 = 1$ True

The solution set is $\{-20\}$.

55. $\frac{3x}{4} + \frac{5x}{2} = 13$
Multiply each side by the LCD, 4.

$$4\left(\frac{3x}{4} + \frac{5x}{2}\right) = 4(13)$$

$$4\left(\frac{3x}{4}\right) + 4\left(\frac{5x}{2}\right) = 4(13) \quad \text{Distributive property}$$

$$3x + 10x = 52$$

$$13x = 52 \quad \text{Combine terms.}$$

$$x = 4 \quad \text{Divide by 13.}$$

Check $x = 4$: $3 + 10 = 13$ True

The solution set is $\{4\}$.

56. $\frac{8x}{3} - \frac{2x}{4} = -13$
Multiply each side by the LCD, 12.

$$12\left(\frac{8x}{3} - \frac{2x}{4}\right) = 12(-13)$$

$$12\left(\frac{8x}{3}\right) - 12\left(\frac{2x}{4}\right) = 12(-13) \quad \text{Distributive property}$$

$$32x - 6x = -156$$

$$26x = -156$$

$$x = -6 \quad \text{Divide by 26.}$$

Check $x = -6$: $-16 + 3 = -13$ True

The solution set is $\{-6\}$.

57. $\frac{x-10}{5} + \frac{2}{5} = -\frac{x}{3}$
Multiply both sides by the LCD, 15.

$$15\left(\frac{x-10}{5} + \frac{2}{5}\right) = 15\left(-\frac{x}{3}\right)$$

$$3(x-10) + 3(2) = -5x$$

$$3x - 30 + 6 = -5x$$

$$8x = 24$$

$$x = \frac{24}{8} = 3$$

Check $x = 3$: $-\frac{7}{5} + \frac{2}{5} = -1$ True

The solution set is $\{3\}$.

58. $\frac{2r-3}{7} + \frac{3}{7} = -\frac{r}{3}$
Multiply both sides by the LCD, 21.

$$21\left(\frac{2r-3}{7} + \frac{3}{7}\right) = 21\left(-\frac{r}{3}\right)$$

$$3(2r-3) + 3(3) = 7(-r)$$

$$6r - 9 + 9 = -7r$$

$$6r = -7r$$

$$13r = 0$$

$$r = \frac{0}{13} = 0$$

Check $r = 0$: $-\frac{3}{7} + \frac{3}{7} = 0$ True

The solution set is $\{0\}$.

59. $\frac{3x-1}{4} + \frac{x+3}{6} = 3$
Multiply each side by the LCD, 12.

$$12\left(\frac{3x-1}{4} + \frac{x+3}{6}\right) = 12(3)$$

$$3(3x-1) + 2(x+3) = 36$$

$$9x - 3 + 2x + 6 = 36$$

$$11x + 3 = 36$$

$$11x = 33$$

$$x = 3$$

Check $x = 3$: $2 + 1 = 3$ True

The solution set is $\{3\}$.

60. $\frac{3x+2}{7} - \frac{x+4}{5} = 2$
Multiply each side by the LCD, 35.

$$35\left(\frac{3x+2}{7} - \frac{x+4}{5}\right) = 35(2)$$

$$5(3x+2) - 7(x+4) = 70$$

$$15x + 10 - 7x - 28 = 70$$

$$8x - 18 = 70$$

$$8x = 88$$

$$x = 11$$

Check $x = 11$: $5 - 3 = 2$ True

The solution set is $\{11\}$.

61. $\frac{4t+1}{3} = \frac{t+5}{6} + \frac{t-3}{6}$
Multiply both sides by the LCD, 6.

$$6\left(\frac{4t+1}{3}\right) = 6\left(\frac{t+5}{6} + \frac{t-3}{6}\right)$$

$$2(4t+1) = (t+5) + (t-3)$$

$$8t + 2 = 2t + 2$$

$$6t = 0$$

$$t = 0$$

Check $t = 0$: $\frac{1}{3} = \frac{5}{6} - \frac{3}{6}$ True

The solution set is $\{0\}$.

$$62. \quad \frac{2x+5}{5} = \frac{3x+1}{2} + \frac{-x+7}{2}$$

Multiply both sides by the LCD, 10.

$$10\left(\frac{2x+5}{5}\right) = 10\left(\frac{3x+1}{2} + \frac{-x+7}{2}\right)$$

$$2(2x+5) = 5(3x+1) + 5(-x+7)$$

$$4x+10 = 15x+5-5x+35$$

$$4x+10 = 10x+40$$

$$-6x = 30$$

$$x = \frac{30}{-6} = -5$$

Check $x = -5$: $-1 = -7 + 6$ True

The solution set is $\{-5\}$.

$$63. \quad 0.05x + 0.12(x + 5000) = 940$$

Multiply both sides by 100.

$$5x + 12(x + 5000) = 100(940)$$

$$5x + 12x + 60,000 = 94,000$$

$$17x = 34,000$$

$$x = 2000$$

Check $x = 2000$: $100 + 840 = 940$ True

The solution set is $\{2000\}$.

$$64. \quad 0.09k + 0.13(k + 300) = 61$$

Multiply both sides by 100.

$$100[0.09k + 0.13(k + 300)] = 100(61)$$

$$100(0.09k) + 100(0.13)(k + 300) = 6100$$

$$9k + 13(k + 300) = 6100$$

$$9k + 13k + 3900 = 6100$$

$$22k = 2200$$

$$k = \frac{2200}{22} = 100$$

Check $k = 100$: $9 + 52 = 61$ True

The solution set is $\{100\}$.

$$65. \quad 0.02(50) + 0.08r = 0.04(50 + r)$$

Multiply both sides by 100.

$$2(50) + 8r = 4(50 + r)$$

$$100 + 8r = 200 + 4r$$

$$4r = 100$$

$$r = 25$$

Check $r = 25$: $1 + 2 = 3$ True

The solution set is $\{25\}$.

$$66. \quad 0.20(14,000) + 0.14t = 0.18(14,000 + t)$$

Multiply both sides by 100.

$$100[0.20(14,000) + 0.14t] = 100[0.18(14,000 + t)]$$

$$20(14,000) + 14t = 18(14,000 + t)$$

$$280,000 + 14t = 252,000 + 18t$$

$$28,000 = 4t$$

$$t = 7000$$

Check $t = 7000$: $2800 + 980 = 3780$ True

The solution set is $\{7000\}$.

$$67. \quad 0.05x + 0.10(200 - x) = 0.45x$$

Multiply both sides by 100.

$$5x + 10(200 - x) = 45x$$

$$5x + 2000 - 10x = 45x$$

$$2000 - 5x = 45x$$

$$2000 = 50x$$

$$40 = x$$

Check $x = 40$: $2 + 16 = 18$ True

The solution set is $\{40\}$.

$$68. \quad 0.08x + 0.12(260 - x) = 0.48x$$

Multiply both sides by 100.

$$8x + 12(260 - x) = 48x$$

$$8x + 3120 - 12x = 48x$$

$$-4x + 3120 = 48x$$

$$3120 = 52x$$

$$x = \frac{3120}{52} = 60$$

Check $x = 60$: $4.8 + 24 = 28.8$ True

The solution set is $\{60\}$.

$$69. \quad 0.006(x + 2) = 0.007x + 0.009$$

Multiply each side by 1000.

$$6(x + 2) = 7x + 9$$

$$6x + 12 = 7x + 9$$

$$3 = x$$

Check $x = 3$: $0.03 = 0.021 + 0.009$ True

The solution set is $\{3\}$.

$$70. \quad 0.004x + 0.006(50 - x) = 0.004(68)$$

Multiply each side by 1000.

$$4x + 6(50 - x) = 4(68)$$

$$4x + 300 - 6x = 272$$

$$-2x + 300 = 272$$

$$-2x = -28$$

$$x = 14$$

Check $x = 14$: $0.056 + 0.216 = 0.272$ True

The solution set is $\{14\}$.

$$71. \quad 2L + 2W; \quad L = 10, W = 8$$

$$2L + 2W = 2(10) + 2(8)$$

$$= 20 + 16 = 36$$

$$72. \quad rt; \quad r = 0.15, t = 3$$

$$rt = 0.15(3) = 0.45$$

$$73. \quad \frac{1}{3}Bh; \quad B = 27, h = 8$$

$$\frac{1}{3}Bh = \frac{1}{3}(27)(8)$$

$$= 9(8) = 72$$

74. $p r t$; $p = 8000$, $r = 0.06$, $t = 2$

$$\begin{aligned} p r t &= 8000(0.06)(2) \\ &= 480(2) = 960 \end{aligned}$$

75. $\frac{5}{9}(F - 32)$; $F = 122$

$$\begin{aligned} \frac{5}{9}(F - 32) &= \frac{5}{9}(122 - 32) \\ &= \frac{5}{9}(90) = 50 \end{aligned}$$

76. $\frac{9}{5}C + 32$; $C = 60$

$$\begin{aligned} \frac{9}{5}C + 32 &= \frac{9}{5}(60) + 32 \\ &= 108 + 32 = 140 \end{aligned}$$

77. $\frac{1}{2}h(b + B)$; $B = 9$, $b = 4$, $h = 3$

$$\begin{aligned} \frac{1}{2}h(b + B) &= \frac{1}{2}(3)(4 + 9) \\ &= \frac{3}{2}(13) = \frac{39}{2}, \text{ or } 19\frac{1}{2} \end{aligned}$$

78. $\frac{1}{2}bh$; $b = 21$, $h = 7$

$$\begin{aligned} \frac{1}{2}bh &= \frac{1}{2}(21)(7) \\ &= \frac{21}{2}(7) = \frac{147}{2}, \text{ or } 73\frac{1}{2} \end{aligned}$$

2.2 Formulas

2.2 Classroom Examples

1. Solve $m = 2k + 3b$ for k .

$$m - 3b = 2k + 3b - 3b \quad \text{Subtract } 3b.$$

$$m - 3b = 2k$$

$$\frac{m - 3b}{2} = \frac{2k}{2} \quad \text{Divide by 2.}$$

$$\frac{m - 3b}{2} = k, \text{ or } k = \frac{m - 3b}{2}$$

2. Solve $y = \frac{1}{2}(x + 3)$ for x .

$$2y = x + 3 \quad \text{Multiply by 2.}$$

$$2y - 3 = x, \text{ or } x = 2y - 3 \quad \text{Subtract 3.}$$

3. Solve $A = 2HW + 2LW + 2LH$ for W .

$$A - 2LH = 2HW + 2LW$$

$$\text{Subtract } 2LH.$$

$$A - 2LH = W(2H + 2L)$$

$$\text{Distributive property}$$

$$\frac{A - 2LH}{2H + 2L} = \frac{W(2H + 2L)}{2H + 2L}$$

$$\frac{A - 2LH}{2H + 2L} = W, \text{ or } W = \frac{A - 2LH}{2H + 2L}$$

4. Use $d = rt$. Solve for t .

$$\frac{d}{r} = \frac{rt}{r} \quad \text{Divide by } r.$$

$$\frac{d}{r} = t, \text{ or } t = \frac{d}{r}$$

Now substitute $d = 500$ and $r = 25$.

$$t = \frac{500}{25} = 20$$

The time is 20 hours.

5. (a) The given amount of mixture is 20 oz. The part that is oil is 1 oz. Thus, the percent of oil is

$$\frac{\text{partial amount}}{\text{whole amount}} = \frac{1}{20} = 0.05 = 5\%.$$

(b) Let x represent the amount of commission earned.

$$\begin{aligned} \frac{x}{12,000} &= 0.08 & \frac{\text{partial}}{\text{whole}} &= \text{percent} \\ x &= 0.08(12,000) & \text{Multiply by 12,000.} \\ x &= 960 \end{aligned}$$

The salesman earns \$960.

6. Let x represent the amount spent on pet supplies/medicine.

$$\begin{aligned} \frac{x}{35.9} &= 0.245 & 24.5\% &= 0.245 \\ x &= 0.245(35.9) & \text{Multiply by 35.9.} \\ x &= 8.7955 \end{aligned}$$

Therefore, about \$8.8 billion was spent on pet supplies/medicine.

2.2 Section Exercises

1. (a) $x = \frac{5x + 8}{3}$

$$3x = 3\left(\frac{5x + 8}{3}\right)$$

$$3x = 5x + 8$$

(b) $t = \frac{bt + k}{c}$ ($c \neq 0$)

$$ct = c\left(\frac{bt + k}{c}\right)$$

$$ct = bt + k$$

2. (a) $3x - 5x = 5x + 8 - 5x$

$$3x - 5x = 8$$

(b) $ct - bt = bt + k - bt$

$$ct - bt = k$$

3. Use the distributive property in each case.

(a) $-2x = 8$

(b) $(c - b)t = k$

4. (a) $\frac{-2x}{-2} = \frac{8}{-2}$

$$x = -4$$

(b) $\frac{(c - b)t}{c - b} = \frac{k}{c - b}$

$$t = \frac{k}{c - b}$$

5. The restriction $b \neq c$ must be applied. If $b = c$, the denominator becomes 0 and division by 0 is undefined.

6. To solve an equation for a particular variable, such as solving the second equation for t , go through the same steps as you would in solving for x in the first equation. Treat all other variables as constants.

7. Solve $I = prt$ for r .

$$\begin{aligned} I &= prt \\ \frac{I}{pt} &= \frac{prt}{pt} \\ \frac{I}{pt} &= r, \text{ or } r = \frac{I}{pt} \end{aligned}$$

8. Solve $d = rt$ for t .

$$\begin{aligned} \frac{d}{r} &= \frac{rt}{r} && \text{Divide by } r. \\ \frac{d}{r} &= t, \text{ or } t = \frac{d}{r} \end{aligned}$$

9. Solve $P = 2L + 2W$ for L .

$$\begin{aligned} P &= 2L + 2W \\ P - 2W &= 2L \\ \frac{P - 2W}{2} &= \frac{2L}{2} \\ \frac{P - 2W}{2} &= L, \text{ or } L = \frac{P}{2} - W \end{aligned}$$

10. Solve $A = bh$ for b .

$$\frac{A}{h} = b \quad \text{Divide by } h.$$

11. (a) Solve for $V = LWH$ for W .

$$\begin{aligned} V &= LWH \\ \frac{V}{LH} &= \frac{LWH}{LH} \\ \frac{V}{LH} &= W, \text{ or } W = \frac{V}{LH} \end{aligned}$$

- (b) Solve for $V = LWH$ for H .

$$\begin{aligned} V &= LWH \\ \frac{V}{LW} &= \frac{LWH}{LW} \\ \frac{V}{LW} &= H, \text{ or } H = \frac{V}{LW} \end{aligned}$$

12. (a) Solve $P = a + b + c$ for b .

$$\begin{aligned} P - (a + c) &= a + b + c - (a + c) \\ &\text{Subtract } (a + c). \\ P - a - c &= b \end{aligned}$$

- (b) Solve $P = a + b + c$ for c .

$$\begin{aligned} P - (a + b) &= a + b + c - (a + b) \\ &\text{Subtract } (a + b). \\ P - a - b &= c \end{aligned}$$

13. Solve $C = 2\pi r$ for r .

$$\begin{aligned} C &= 2\pi r \\ \frac{C}{2\pi} &= \frac{2\pi r}{2\pi} && \text{Divide by } 2\pi. \\ \frac{C}{2\pi} &= r \end{aligned}$$

14. Solve $A = \frac{1}{2}bh$ for h .

$$\begin{aligned} \frac{2A}{b} &= \frac{2}{b} \left(\frac{1}{2}bh \right) && \text{Multiply by 2;} \\ &&& \text{divide by } b. \\ \frac{2A}{b} &= h \end{aligned}$$

15. (a) Solve $A = \frac{1}{2}h(b + B)$ for h .

$$\begin{aligned} 2A &= h(b + B) && \text{Multiply by 2.} \\ \frac{2A}{b + B} &= h && \text{Divide by } b + B. \end{aligned}$$

- (b) Solve $A = \frac{1}{2}h(b + B)$ for B .

$$\begin{aligned} 2A &= h(b + B) \text{ or } 2A = hb + hB \\ \frac{2A}{h} &= b + B && 2A - hb = hB \\ \frac{2A}{h} - b &= B, \text{ or } \frac{2A - hb}{h} = B \end{aligned}$$

16. Solve $S = 2\pi rh + 2\pi r^2$ for h .

$$\begin{aligned} S - 2\pi r^2 &= 2\pi rh && \text{Subtract } 2\pi r^2. \\ \frac{S - 2\pi r^2}{2\pi r} &= \frac{2\pi rh}{2\pi r} && \text{Divide by } 2\pi r. \\ \frac{S - 2\pi r^2}{2\pi r} &= h, \text{ or } \frac{S}{2\pi r} - r = h \end{aligned}$$

17. Solve $F = \frac{9}{5}C + 32$ for C .

$$\begin{aligned} F - 32 &= \frac{9}{5}C \\ \frac{5}{9}(F - 32) &= \frac{5}{9} \left(\frac{9}{5}C \right) \\ \frac{5}{9}(F - 32) &= C \end{aligned}$$

18. Solve $C = \frac{5}{9}(F - 32)$ for F .

$$\begin{aligned} \frac{9}{5}C &= \frac{9}{5} \cdot \frac{5}{9}(F - 32) && \text{Multiply by } \frac{9}{5}. \\ \frac{9}{5}C &= F - 32 \\ \frac{9}{5}C + 32 &= F && \text{Add 32.} \end{aligned}$$

19. $A = \frac{1}{2}bh$

$$2A = 2 \left(\frac{1}{2}bh \right) \quad \text{Multiply by 2.}$$

$$2A = bh$$

$$\frac{2A}{b} = \frac{bh}{b} \quad \text{Divide by } b.$$

$$\frac{2A}{b} = h$$

$$\frac{2A}{b} = \frac{2}{1} \cdot \frac{A}{b}$$

$$= 2 \left(\frac{A}{b} \right) \quad \text{This choice is A.}$$

$$= 2A \left(\frac{1}{b} \right) \quad \text{This is choice B.}$$

continued

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To get choice C, divide $A = \frac{1}{2}bh$ by $\frac{1}{2}b$.

$$\frac{A}{\frac{1}{2}b} = \frac{\frac{1}{2}bh}{\frac{1}{2}b} \text{ gives us } h = \frac{A}{\frac{1}{2}b}.$$

Choice D, $h = \frac{\frac{1}{2}A}{b}$, can be multiplied by $\frac{2}{2}$ on the right side to get $h = \frac{A}{2b}$, so it is *not* equivalent to $h = \frac{2A}{b}$. Therefore, the correct answer is D.

20. The formula is not solved for L because L should not appear on each side of the final equation. Solve for L as follows:

$$\begin{aligned} A &= 2HW + 2LW + 2LH \\ A - 2HW &= 2LW + 2LH \\ A - 2HW &= L(2W + 2H) \\ \frac{A - 2HW}{2W + 2H} &= L \end{aligned}$$

21. Solve $2k + ar = r - 3y$ for r .
Get the " r -terms" on one side and the other terms on the other side.

$$\begin{aligned} ar - r &= -2k - 3y \\ (a - 1)r &= -2k - 3y && \text{Distributive property} \\ r &= \frac{-2k - 3y}{a - 1} && \text{Divide by } a - 1. \end{aligned}$$

The answer can also be written as

$$r = \frac{2k + 3y}{1 - a},$$

which would occur if you took the r -terms to the right side in your first step.

22. Solve $4s + 7p = tp - 7$ for p .

$$\begin{aligned} 7p - tp &= -4s - 7 \\ (7 - t)p &= -4s - 7 \\ p &= \frac{-4s - 7}{7 - t}, \text{ or } p = \frac{4s + 7}{t - 7} \end{aligned}$$

23. Solve $w = \frac{3y - x}{y}$ for y .

$$\begin{aligned} wy &= 3y - x && \text{Multiply by } y. \\ x &= 3y - wy && \text{Get } y\text{-terms on one side.} \\ x &= (3 - w)y && \text{Distributive property} \\ \frac{x}{3 - w} &= y && \text{Divide by } 3 - w. \end{aligned}$$

Equivalently, we have

$$y = \frac{-x}{w - 3}.$$

24. Solve $c = \frac{-2t + 4}{t}$ for t .

$$\begin{aligned} ct &= -2t + 4 \\ ct + 2t &= 4 \\ (c + 2)t &= 4 \\ t &= \frac{4}{c + 2} \text{ or } t = \frac{-4}{-c - 2} \end{aligned}$$

25. Solve $d = rt$ for t .

$$t = \frac{d}{r}$$

To find t , substitute $d = 500$ and $r = 135.173$.

$$t = \frac{500}{135.173} \approx 3.699$$

His time was about 3.699 hours.

26. Solve $d = rt$ for t .

$$t = \frac{d}{r}$$

Replace d by 450 and r by 138.518.

$$t = \frac{450}{138.518} \approx 3.249$$

His time was about 3.249 hours.

27. Solve $d = rt$ for r .

$$\begin{aligned} r &= \frac{d}{t} \\ r &= \frac{520}{10} = 52 && \text{Let } d = 520, t = 10. \end{aligned}$$

Her rate was 52 mph.

28. Solve $d = rt$ for t .

$$\begin{aligned} t &= \frac{d}{r} \\ t &= \frac{10,500}{500} = 21 && \text{Let } d = 10,500, r = 500. \end{aligned}$$

The travel time is 21 hr.

29. Use the formula $F = \frac{9}{5}C + 32$.

$$\begin{aligned} F &= \frac{9}{5}(40) + 32 && \text{Let } C = 40. \\ &= 72 + 32 \\ &= 104 \end{aligned}$$

The corresponding temperature is 104°F.

30. Use the formula $C = \frac{5}{9}(F - 32)$.

$$\begin{aligned} C &= \frac{5}{9}(-13 - 32) && \text{Let } F = -13. \\ &= \frac{5}{9}(-45) \\ &= -25 \end{aligned}$$

The corresponding temperature is about -25°C .

31. Solve $P = 4s$ for s .

$$s = \frac{P}{4}$$

To find s , substitute 920 for P .

$$s = \frac{920}{4} = 230$$

The length of each side is 230 m.

32. Use $V = \pi r^2 h$.
Replace r by $\frac{35}{2} = 17.5$ and h by 588.

$$\begin{aligned} V &= \pi(17.5^2)(588) \\ &\approx 565,722.3 \end{aligned}$$

To the nearest whole number, the volume is 565,722 ft³.

33. Use the formula $C = 2\pi r$.

$$\begin{aligned} 480\pi &= 2\pi r & \text{Let } C &= 480\pi. \\ \frac{480\pi}{2\pi} &= r & \text{Divide by } 2\pi. \end{aligned}$$

So the radius of the circle is 240 inches and the diameter is twice that length, that is, 480 inches.

34. $d = 2r = 2(2.5) = 5$
The diameter is 5 inches.
 $C = 2\pi r = 2\pi(2.5) = 5\pi$
The circumference is 5π inches.

35. Use $V = LWH$.
Let $V = 187$, $L = 11$, and $W = 8.5$.

$$\begin{aligned} 187 &= 11(8.5)H \\ 187 &= 93.5H \\ 2 &= H & \text{Divide by } 93.5. \end{aligned}$$

The ream is 2 inches thick.

36. Use $V = LWH$.
Let $V = 238$, $W = 8.5$, and $H = 2$.

$$\begin{aligned} 238 &= L(8.5)(2) \\ 238 &= L(17) \\ 14 &= L & \text{Divide by } 17. \end{aligned}$$

The length of a legal sheet of paper is 14 inches.

37. The mixture is 36 oz and that part which is alcohol is 9 oz. Thus, the percent of alcohol is

$$\frac{9}{36} = \frac{1}{4} = \frac{25}{100} = 25\%.$$

The percent of water is

$$100\% - 25\% = 75\%.$$

38. Let x = the amount of pure acid in the mixture.
Then x can be found by multiplying the total amount of the mixture by the percent of acid given as a decimal (0.35).

$$x = 40(0.35) = 14$$

There are 14 L of pure acid. Since there are 40 L altogether, there are $40 - 14$, or 26 L of pure water in the mixture.

39. Find what percent \$6300 is of \$210,000.

$$\frac{6300}{210,000} = 0.03 = 3\%$$

The agent received a 3% rate of commission.

40. Solve $I = prt$ for r .

$$\begin{aligned} r &= \frac{I}{pt} \\ r &= \frac{221}{3400(1)} \\ &= 0.065 = 6.5\% \end{aligned}$$

The interest rate on this deposit is 6.5%.

In Exercises 41–44, use the rule of 78.

$$u = f \cdot \frac{k(k+1)}{n(n+1)}$$

41. Substitute 700 for f , 4 for k , and 36 for n .

$$\begin{aligned} u &= 700 \cdot \frac{4(4+1)}{36(36+1)} \\ &= 700 \cdot \frac{4(5)}{36(37)} \approx 10.51 \end{aligned}$$

The unearned interest is \$10.51.

42. Substitute 600 for f , 12 for k , and 36 for n .

$$\begin{aligned} u &= 600 \cdot \frac{12(12+1)}{36(36+1)} \\ &= 600 \cdot \frac{12(13)}{36(37)} \approx 70.27 \end{aligned}$$

The unearned interest is \$70.27.

43. Substitute 380.50 for f , 8 for k , and 24 for n .

$$\begin{aligned} u &= (380.50) \cdot \frac{8(8+1)}{24(24+1)} \\ &= (380.50) \cdot \frac{8(9)}{24(25)} \approx 45.66 \end{aligned}$$

The unearned interest is \$45.66.

44. Substitute 450 for f , 9 for k , and 24 for n .

$$\begin{aligned} u &= 450 \cdot \frac{9(9+1)}{24(24+1)} \\ &= 450 \cdot \frac{9(10)}{24(25)} \approx 67.50 \end{aligned}$$

The unearned interest is \$67.50.

45. (a) Detroit:

$$\text{Pct.} = \frac{W}{W+L} = \frac{19}{19+9} = \frac{19}{28} \approx .679$$

(b) Cleveland:

$$\text{Pct.} = \frac{W}{W+L} = \frac{14}{14+13} = \frac{14}{27} \approx .519$$

(c) Minnesota:

$$\text{Pct.} = \frac{W}{W+L} = \frac{11}{11+16} = \frac{11}{27} \approx .407$$

(d) Kansas City:

$$\text{Pct.} = \frac{W}{W+L} = \frac{5}{5+20} = \frac{5}{25} = .200$$

46. (a) Cincinnati:

$$\text{Pct.} = \frac{W}{W+L} = \frac{19}{19+9} = \frac{19}{28} \approx .679$$

(b) Houston:

$$\text{Pct.} = \frac{W}{W+L} = \frac{18}{18+9} = \frac{18}{27} \approx .667$$

(c) Chicago:

$$\text{Pct.} = \frac{W}{W+L} = \frac{14}{14+12} = \frac{14}{26} \approx .538$$

(d) Pittsburgh

$$\text{Pct.} = \frac{W}{W+L} = \frac{8}{8+21} = \frac{8}{29} \approx .276$$

47. $0.34(242,070) = 82,303.80$

To the nearest dollar, \$82,304 will be spent to provide housing.

48. $0.07(242,070) = 16,944.90$

To the nearest dollar, \$16,945 will be spent for health care.

49. Since 14% is twice as much as 7%, the cost for transportation will be $2(\$16,945) = \$33,890$.50. $\frac{\$41,000}{\$242,070} \approx 0.1694$

So the food cost is about 17%, which agrees with the percent shown in the graph.

51. In 1982, 3% of the television audience watched basic cable.

$$\begin{aligned} &3\% \text{ of } 50,000 \\ &= 0.03(50,000) \\ &= 1500 \end{aligned}$$

52. $21\% \text{ of } 110,000 = 0.21(110,000) = 23,100$

53. In 2004, 54% of the television audience watched cable.

$$\begin{aligned} &54\% \text{ of } 35,000 \\ &= 0.54(35,000) \\ &= 18,900 \end{aligned}$$

54. $6\% \text{ of } 65,000 = 0.06(65,000) = 3900$

$$\begin{aligned} 55. \quad &4x + 4(x+7) = 124 \\ &4x + 4x + 28 = 124 \\ &8x + 28 = 124 \\ &8x = 96 \\ &x = \frac{96}{8} = 12 \end{aligned}$$

The solution set is $\{12\}$.

$$\begin{aligned} 56. \quad &x + 0.20x = 66 \\ &1.2x = 66 \\ &x = \frac{66}{1.2} = 55 \end{aligned}$$

The solution set is $\{55\}$.

$$\begin{aligned} 57. \quad &\frac{5}{3} + \frac{2}{3}x = 2 \\ &5 + 2x = 6 \quad \text{Multiply by 3.} \\ &2x = 1 \\ &x = \frac{1}{2} \end{aligned}$$

The solution set is $\{\frac{1}{2}\}$.

$$\begin{aligned} 58. \quad &0.07x + 0.05(9000 - x) = 510 \\ &7x + 5(9000 - x) = 510 \\ &\quad \text{Multiply by 100.} \\ &7x + 45,000 - 5x = 51,000 \\ &2x + 45,000 = 51,000 \\ &2x = 6000 \\ &x = 3000 \end{aligned}$$

The solution set is $\{3000\}$.

$$\begin{aligned} 59. \quad &2.4 + 0.4x = 0.25(6 + x) \\ &240 + 40x = 25(6 + x) \\ &\quad \text{Multiply by 100.} \\ &240 + 40x = 150 + 25x \\ &15x = -90 \\ &x = -6 \end{aligned}$$

The solution set is $\{-6\}$.

$$\begin{aligned} 60. \quad &5(x-2) + 8x = 16 \\ &5x - 10 + 8x = 16 \\ &13x - 10 = 16 \\ &13x = 26 \\ &x = 2 \end{aligned}$$

The solution set is $\{2\}$.

61. "The product of -3 and 5 , divided by 1 less than 6 " is translated and evaluated as follows:

$$\frac{-3(5)}{6-1} = \frac{-15}{5} = -3$$

62. "Half of -18 , added to the reciprocal of $\frac{1}{5}$ " is translated and evaluated as follows:

$$\frac{1}{2}(-18) + \frac{1}{\frac{1}{5}} = -9 + 5 = -4$$

63. "The sum of 6 and -9 , multiplied by the additive inverse of 2" is translated and evaluated as follows:

$$[6 + (-9)](-2) = -3(-2) = 6$$

64. "The product of -2 and 4, added to the product of -9 and -3 " is translated and evaluated as follows:

$$-2(4) + (-9)(-3) = -8 + 27 = 19$$

2.3 Applications of Linear Equations

2.3 Classroom Examples

1. (a) The sum of a number and 6 is 28.

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ x + 6 & = & 28 \end{array}$$

An equation is $x + 6 = 28$.

- (b) The product of a number and 7 is twice the number plus 12.

$$\begin{array}{ccccccc} \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ 7x & = & 2x & + & 12 & & \end{array}$$

An equation is $7x = 2x + 12$.

- (c) The quotient of a number and 6, added to twice the number, is 7.

$$\begin{array}{ccccccc} \downarrow & & \downarrow & \downarrow & \downarrow & \downarrow & \\ \frac{x}{6} & + & 2x & = & 7 & & \end{array}$$

An equation is $\frac{x}{6} + 2x = 7$, or equivalently, $2x + \frac{x}{6} = 7$.

2. (a) $5x - 3(x + 2) = 7$ is an *equation* because it has an equals sign.
 (b) $5x - 3(x + 2)$ is an *expression* because there is no equals sign.

3. *Step 2*

The length and perimeter are given in terms of the width W . The length L is 5 cm more than the width, so

$$L = W + 5.$$

The perimeter P is 5 times the width, so

$$P = 5W.$$

Step 3

Use the formula for perimeter of a rectangle.

$$P = 2L + 2W$$

$$5W = 2(W + 5) + 2W \quad P = 5W; L = W + 5$$

Step 4

Solve the equation.

$$5W = 2W + 10 + 2W \quad \text{Distributive property}$$

$$5W = 4W + 10 \quad \text{Combine terms.}$$

$$W = 10 \quad \text{Subtract } 4W.$$

Step 5

The width is 10 and the length is

$$L = W + 5 = 10 + 5 = 15.$$

The rectangle is 10 cm by 15 cm.

Step 6

15 is 5 more than 10 and $P = 2(10) + 2(15) = 50$ is five times 10.

4. *Step 2*

Let x = the number of RBIs for Jones.

Then $x + 20$ = the number of RBIs for Ortiz.

Step 3

The sum of their RBIs is 276, so an equation is

$$x + (x + 20) = 276.$$

Step 4

Solve the equation.

$$2x + 20 = 276$$

$$2x = 256 \quad \text{Subtract 20.}$$

$$x = 128 \quad \text{Divide by 2.}$$

Step 5

Jones had 128 RBIs and Ortiz had

$$128 + 20 = 148 \text{ RBIs.}$$

Step 6

148 is 20 more than 128, and the sum of 128 and 148 is 276.

5. Let x = the number.

187.5 is 125% of some number

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 187.5 & = & 125\%(x) \\ 187.5 & = & 125(0.01)x \\ 187.5 & = & 1.25x \\ x & = & \frac{187.5}{1.25} = 150 \end{array}$$

The number is 150.

6. Let x = the amount invested at 5%.
Then $34,000 - x$ = the amount invested at 4%.

Use $I = prt$ with $t = 1$.

Make a table to organize the information.

Principal	Rate (as a decimal)	Interest
x	0.05	$0.05x$
$34,000 - x$	0.04	$0.04(34,000 - x)$
34,000	← Totals →	1545

The last column gives the equation.

$$0.05x + 0.04(34,000 - x) = 1545$$

$$0.05x + 1360 - 0.04x = 1545 \quad \text{Distributive property}$$

$$0.01x + 1360 = 1545 \quad \text{Combine terms.}$$

$$0.01x = 185 \quad \text{Subtract 1360.}$$

$$x = 18,500 \quad \text{Divide by 0.01.}$$

The man invested \$18,500 at 5% and
 $\$34,000 - \$18,500 = \$15,500$ at 4%.

7. Let x = the amount of \$8 per lb candy. Then
 $x + 100$ = the amount of \$7 per lb candy.

Make a table to organize the information.

Number of Pounds	Price per Pound	Value
x	\$8	$8x$
100	\$4	400
$x + 100$	\$7	$7(x + 100)$

The last column gives the equation.

$$8x + 400 = 7(x + 100)$$

$$8x + 400 = 7x + 700 \quad \text{Distributive property}$$

$$x + 400 = 700 \quad \text{Subtract } 7x.$$

$$x = 300 \quad \text{Subtract 400.}$$

300 lb of candy worth \$8 per lb should be used.

8. Let x = the number of liters of water.

Number of Liters	Percent (as a decimal)	Liters of Pure Antifreeze
x	0% = 0	0
20	50% = 0.50	$0.50(20)$
$x + 20$	40% = 0.40	$0.40(x + 20)$

The last column gives the equation.

$$0 + 0.50(20) = 0.40(x + 20)$$

$$10 = 0.4x + 8$$

$$2 = 0.4x \quad \text{Subtract 8.}$$

$$5 = x \quad \text{Divide by 0.4.}$$

5 L of water are needed.

2.3 Section Exercises

- (a) 12 more than a number $x + 12$

(b) 12 is more than a number. $12 > x$
- (a) 3 less than a number $x - 3$

(b) 3 is less than a number. $3 < x$
- (a) 4 less than a number $x - 4$

(b) 4 is less than a number. $4 < x$
- (a) 6 greater than a number $x + 6$

(b) 6 is greater than a number. $6 > x$
- 20% can be written as
 $0.20 = 0.2 = \frac{20}{100} = \frac{2}{10} = \frac{1}{5}$, so "20% of a number" can be written as $0.20x$, $0.2x$, or $\frac{x}{5}$. We see that "20% of a number" cannot be written as $20x$, choice **D**.
- $13 - x$ is the translation of " x less than 13." The phrase "13 less than a number" is translated $x - 13$.
- Twice a number, decreased by 13 $2x - 13$
- The product of 6 and a number, decreased by 12 $6x - 12$
- 12 increased by three times a number $12 + 3x$
- 12 more than one-half of a number $\frac{1}{2}x + 12$
- The product of 8 and 12 less than a number $8(x - 12)$
- The product of 9 more than a number and 6 less than the number $(x + 9)(x - 6)$
- The quotient of three times a number and 7 $\frac{3x}{7}$
- The quotient of 6 and five times a nonzero number $\frac{6}{5x} (x \neq 0)$
- The sentence "the sum of a number and 6 is -31 " can be translated as

$$x + 6 = -31.$$

$$x = -37 \quad \text{Subtract 6.}$$

The number is -37 .
- The sentence "the sum of a number and -4 is 12" can be translated as

$$x + (-4) = 12.$$

$$x = 16 \quad \text{Add 4.}$$

The number is 16.

17. The sentence "if the product of a number and -4 is subtracted from the number, the result is 9 more than the number" can be translated as

$$\begin{aligned}x - (-4x) &= x + 9. \\x + 4x &= x + 9 \\4x &= 9 \\x &= \frac{9}{4}\end{aligned}$$

The number is $\frac{9}{4}$.

18. The sentence "if the quotient of a number and 6 is added to twice the number, the result is 8 less than the number" can be translated as

$$\begin{aligned}2x + \frac{x}{6} &= x - 8. \\12x + x &= 6x - 48 \quad \text{Multiply by 6.} \\13x &= 6x - 48 \\7x &= -48 \\x &= -\frac{48}{7}\end{aligned}$$

The number is $-\frac{48}{7}$.

19. The sentence "when $\frac{2}{3}$ of a number is subtracted from 12, the result is 10" can be translated as

$$\begin{aligned}12 - \frac{2}{3}x &= 10. \\36 - 2x &= 30 \quad \text{Multiply by 3.} \\-2x &= -6 \quad \text{Subtract 36.} \\x &= 3 \quad \text{Divide by } -2.\end{aligned}$$

The number is 3.

20. The sentence "when 75% of a number is added to 6, the result is 3 more than the number" can be translated as

$$\begin{aligned}6 + 0.75x &= x + 3. \\600 + 75x &= 100x + 300 \quad \text{Multiply by 100.} \\600 - 25x &= 300 \quad \text{Subtract } 100x. \\-25x &= -300 \quad \text{Subtract 600.} \\x &= 12 \quad \text{Divide by } -25.\end{aligned}$$

The number is 12.

21. $5(x + 3) - 8(2x - 6)$ is an *expression* because there is no equals sign.
22. $-7(z + 4) + 13(z - 6)$ has no equals sign, so it is an *expression*.
23. $5(x + 3) - 8(2x - 6) = 12$ has an equals sign, so this represents an *equation*.
24. $-7(z + 4) + 13(z - 6) = 18$ has an equals sign, so it is an *equation*.
25. $\frac{r}{2} - \frac{r + 9}{6} - 8$ is an *expression* because there is no equals sign.

26. $\frac{r}{2} - \frac{r + 9}{6} = 8$ has an equals sign, so it is an *equation*.

27. *Step 1*
We are asked to find the number of patents each university secured.

Step 2

Let x = the number of patents MIT secured.

Then $x - 38$ = the number of patents Stanford secured.

Step 3

A total of 230 patents were secured, so

$$\underline{x} + \underline{x - 38} = 230.$$

Step 4

$$\begin{aligned}2x - 38 &= 230 \\2x &= 268 \\x &= \underline{134}\end{aligned}$$

Step 5

MIT secured 134 patents and Stanford secured $134 - 38 = \underline{96}$ patents.

Step 6

The number of Stanford patents was 38 fewer than the number of MIT patents and the total number of patents was $134 + \underline{96} = \underline{230}$.

28. *Step 1*
We are asked to find the number of book buyers at each type of bookstore.

Step 2

Let x = the number of book buyers at large-chain bookstores. Then

$x - 70$ = the number of book buyers at small-chain/independent bookstores.

Step 3

A total of 442 book buyers shopped at these two types of stores, so

$$\underline{x} + \underline{(x - 70)} = 442.$$

Step 4

$$\begin{aligned}2x - 70 &= 442 \\2x &= 512 \\x &= \underline{256}\end{aligned}$$

Step 5

There were 256 large-chain bookstore shoppers and $256 - 70 = \underline{186}$ small-chain/independent shoppers.

Step 6

The number of large-chain shoppers was 70 more than the number of small-chain/independent shoppers, and the total number of these shoppers was $256 + \underline{186} = \underline{442}$.

29. Step 2

Let W = the width of the base. Then $2W - 65$ is the length of the base.

Step 3

The perimeter of the base is 860 feet. Using

$P = 2L + 2W$ gives us

$$2(2W - 65) + 2W = 860.$$

Step 4

$$4W - 130 + 2W = 860$$

$$6W - 130 = 860$$

$$6W = 990$$

$$W = \frac{990}{6} = 165$$

Step 5

The width of the base is 165 feet and the length of the base is $2(165) - 65 = 265$ feet.

Step 6

$2L + 2W = 2(265) + 2(165) = 530 + 330 = 860$, which is the perimeter of the base.

30. Step 2

Let L = the length of the top floor. Then $\frac{1}{2}L + 20$ is the width of the top floor.

Step 3

The perimeter of the top floor is 520 feet. Using

$P = 2L + 2W$ gives us

$$2L + 2\left(\frac{1}{2}L + 20\right) = 520.$$

Step 4

$$2L + L + 40 = 520$$

$$3L + 40 = 520$$

$$3L = 480$$

$$L = 160$$

Step 5

The length of the top floor is 160 feet and the width of the top floor is $\frac{1}{2}(160) + 20 = 100$ feet.

Step 6

$2L + 2W = 2(160) + 2(100) = 320 + 200 = 520$, which is the perimeter of the top floor.

31. Step 2

Let x = the width of the painting. Then

$x + 5.54$ = the height of the painting.

Step 3

The perimeter of the painting is 108.44 inches. Using

$P = 2L + 2W$ gives us

$$2(x + 5.54) + 2x = 108.44.$$

Step 4

$$2x + 11.08 + 2x = 108.44$$

$$4x + 11.08 = 108.44$$

$$4x = 97.36$$

$$x = 24.34$$

Step 5

The width of the painting is 24.34 inches and the height is $24.34 + 5.54 = 29.88$ inches.

Step 6

29.88 is 5.54 more than 24.34 and

$2(29.88) + 2(24.34) = 108.44$, as required.

32. Step 2

Let W = the width of the rectangle. Then

$W + 12$ = the length of the rectangle.

Step 3

The perimeter, $2L + 2W$, is 16 times the width, so

$$2(W + 12) + 2W = 16(W).$$

Step 4

$$2W + 24 + 2W = 16W$$

$$4W + 24 = 16W$$

$$24 = 12W$$

$$2 = W$$

Step 5

The width of the rectangle is 2 centimeters. The length is $2 + 12 = 14$ centimeters.

Step 6

The length is 12 more than the width. The perimeter is $2(14) + 2(2) = 32$, which is 16 times the width, as required.

33. Step 2

Let x = the length of the middle side. Then the shortest side is $x - 75$ and the longest side is $x + 375$.

Step 3

The perimeter of the Bermuda Triangle is 3075 miles. Using $P = a + b + c$ gives us

$$x + (x - 75) + (x + 375) = 3075.$$

Step 4

$$3x + 300 = 3075$$

$$3x = 2775 \quad \text{Subtract 300.}$$

$$x = 925 \quad \text{Divide by 3.}$$

Step 5

The length of the middle side is 925 miles. The length of the shortest side is $x - 75 = 925 - 75 = 850$ miles. The length of the longest side is $x + 375 = 925 + 375 = 1300$ miles.

Step 6

The answer checks since $925 + 850 + 1300 = 3075$ miles, which is the correct perimeter.

34. *Step 2*

Let x = the length of one of the sides of equal length.

Step 3

The perimeter of the triangle is 931.5 feet. Using $P = a + b + c$ gives us

$$x + x + 438 = 931.5$$

Step 4

$$2x + 438 = 931.5$$

$$2x = 493.5 \quad \text{Subtract 438.}$$

$$x = 246.75 \quad \text{Divide by 2.}$$

Step 5

The two walls are each 246.75 feet long.

Step 6

The answer checks since

$246.75 + 246.75 + 438 = 931.5$, which is the correct perimeter.

35. *Step 2*

Let x = the amount of revenue for Exxon Mobil. Then $x - 24$ is the amount of revenue for Wal-Mart (in billions).

Step 3

The total revenue was \$656 billion, so

$$x + (x - 24) = 656.$$

Step 4

$$2x - 24 = 656$$

$$2x = 680 \quad \text{Add 24.}$$

$$x = 340 \quad \text{Divide by 2.}$$

Step 5

The amount of revenue for Exxon Mobil was \$340 billion. The amount of revenue for Wal-Mart was $x - 24 = 340 - 24 = \$316$ billion.

Step 6

The answer checks since $340 + 316 = \$656$ billion, which is the correct total revenue.

36. *Step 2*

Let x = the number of performances of *Cats*. Then $x - 805$ = the number of performances of *Les Misérables*.

Step 3

There were 14,165 total performances, so

$$x + (x - 805) = 14,165.$$

Step 4

$$2x - 805 = 14,165$$

$$2x = 14,970$$

$$x = 7485$$

Step 5

There were 7485 performances of *Cats* and $7485 - 805 = 6680$ performances of *Les Misérables*.

Step 6

The total number of performances is 14,165, as required.

37. *Step 2*

Let x = the height of the Eiffel Tower.

Then $x - 804$ = the height of the Leaning Tower of Pisa.

Step 3

Together these heights are 1164 ft, so

$$x + (x - 804) = 1164.$$

Step 4

$$2x - 804 = 1164$$

$$2x = 1968$$

$$x = 984$$

Step 5

The height of the Eiffel Tower is 984 feet and the height of the Leaning Tower of Pisa is $984 - 804 = 180$ feet.

Step 6

180 feet is 804 feet shorter than 984 feet and the sum of 180 feet and 984 feet is 1164 feet.

38. *Step 2*

Let x = the Yankees' payroll (in millions).

Then $x - 32.8$ = the Mets' payroll (in millions).

Step 3

The two payrolls totaled \$266.6 million, so

$$x + (x - 32.8) = 266.6$$

Step 4

$$2x - 32.8 = 266.6$$

$$2x = 299.4$$

$$x = 149.7$$

Step 5

In 2003, the Yankees' payroll was \$149.7 million and the Mets' payroll was $149.7 - 32.8 = \$116.9$ million.

Step 6

\$116.9 million is \$32.8 million less than \$149.7 million and the sum of \$116.9 million and \$149.7 million is \$266.6 million.

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39. *Step 2*

Let x = votes for Kerry. Then

$x + 35$ = votes for Bush.

Step 3

There were 537 total electoral votes, so

$$x + (x + 35) = 537.$$

Step 4

$$2x + 35 = 537$$

$$2x = 502$$

$$x = 251$$

Step 5

Kerry received 251 votes, so Bush received

$251 + 35 = 286$ votes.

Step 6

286 is 35 more than 251 and the total is

$251 + 286 = 537$.

40. *Step 2*

Let x = the number of hits Williams got. Then

$x + 276$ = the number of hits Hornsby got.

Step 3

Their base hits totaled 5584, so

$$x + (x + 276) = 5584.$$

Step 4

$$2x + 276 = 5584$$

$$2x = 5308$$

$$x = 2654$$

Step 5

Williams got 2654 base hits, and Hornsby got

$2654 + 276 = 2930$ base hits.

Step 6

2930 is 276 more than 2654 and the total is

$2654 + 2930 = 5584$.

41. Let x = the percent increase.

$$\begin{aligned} x &= \frac{\text{amount of increase}}{\text{base number}} \\ &= \frac{1,186,251 - 817,000}{817,000} = \frac{369,251}{817,000} \\ &\approx 0.452 = 45.2\% \end{aligned}$$

The percent increase was approximately 45.2%.

42. Let x = the percent decrease.

$$\begin{aligned} x &= \frac{\text{amount of decrease}}{\text{base score}} \\ &= \frac{21.0 - 20.8}{21.0} = \frac{0.2}{21.0} \\ &\approx 0.0095 = 0.95\% \end{aligned}$$

The percent decrease was approximately 1%.

43. Let x = the approximate cost in 2005.

Since x is 95% more than the 1995 cost,

$$\begin{aligned} x &= 2811 + 0.95(2811) \\ &= 2811 + 2670.45 \\ &= 5481.45. \end{aligned}$$

To the nearest dollar, the cost was \$5481.

44. Let x = the approximate cost in 2005.

Since x is 73.8% more than the 1995 cost,

$$\begin{aligned} x &= 12,216 + 0.738(12,216) \\ &= 12,216 + 9015.408 \\ &\approx 21,231.41. \end{aligned}$$

To the nearest dollar, the cost was \$21,231.

45. Let x = the 2004 cost. Then

$$\begin{aligned} x + 3.1\%(x) &= 36.78. \\ x + 3.1(0.01)(x) &= 36.78 \\ 1x + 0.031x &= 36.78 \\ 1.031x &= 36.78 \\ x &= \frac{36.78}{1.031} \approx 35.67 \end{aligned}$$

The 2004 cost was \$35.67.

46. Let x = the 1987 cost. Then

$$\begin{aligned} x + 37.5\%(x) &= 36.78. \\ x + 37.5(0.01)(x) &= 36.78 \\ 1x + 0.375x &= 36.78 \\ 1.375x &= 36.78 \\ x &= \frac{36.78}{1.375} \approx 26.75 \end{aligned}$$

The 1987 cost was \$26.75.

47. Let x = the amount of the receipts excluding tax.

Since the sales tax is 9% of x , the total amount is

$$\begin{aligned} x + 0.09x &= 2725 \\ 1x + 0.09x &= 2725 \\ 1.09x &= 2725 \\ x &= \frac{2725}{1.09} = 2500 \end{aligned}$$

Thus, the tax was $0.09(2500) = \$225$.

48. Let x = the amount of commission. Since x is 6% of the selling price,

$$x = 0.06(159,000) = 9540.$$

So after the agent was paid, he had

$159,000 - 9540 = \$149,460$.

49. Let x = the amount invested at 3%. Then

$12,000 - x$ = the amount invested at 4%.

Complete the table. Use $I = prt$ with $t = 1$.

Principal	Rate (as a decimal)	Interest
x	0.03	$0.03x$
$12,000 - x$	0.04	$0.04(12,000 - x)$
12,000	← Totals →	440

The last column gives the equation.

$$\begin{array}{rcl} \text{Interest} & + & \text{interest} & = & \text{total} \\ \text{at 3\%} & & \text{at 4\%} & & \text{interest.} \\ 0.03x & + & 0.04(12,000 - x) & = & 440 \end{array}$$

$$3x + 4(12,000 - x) = 44,000 \quad \text{Multiply by 100.}$$

$$\begin{aligned} 3x + 48,000 - 4x &= 44,000 \\ -x &= -4000 \\ x &= 4000 \end{aligned}$$

He should invest \$4000 at 3% and
12,000 - 4000 = \$8000 at 4%.

Check \$4000 @ 3% = \$120 and
\$8000 @ 4% = \$320; \$120 + \$320 = \$440.

50. Let x = the amount invested at 2%. Then
60,000 - x = the amount invested at 3%.
Complete the table. Use $I = prt$ with $t = 1$.

Principal	Rate (as a decimal)	Interest
x	0.02	$0.02x$
$60,000 - x$	0.03	$0.03(60,000 - x)$
60,000	← Totals →	1600

The last column gives the equation.

$$\begin{array}{rcl} \text{Interest} & + & \text{interest} & = & \text{total} \\ \text{at 2\%} & & \text{at 3\%} & & \text{interest.} \\ 0.02x & + & 0.03(60,000 - x) & = & 1600 \end{array}$$

$$2x + 3(60,000 - x) = 160,000 \quad \text{Multiply by 100.}$$

$$\begin{aligned} 2x + 180,000 - 3x &= 160,000 \\ -x &= -20,000 \\ x &= 20,000 \end{aligned}$$

She invested \$20,000 at 2% and
60,000 - x = 60,000 - 20,000 = \$40,000 at 3%.

Check \$20,000 @ 2% = \$400 and
\$40,000 @ 3% = \$1200; \$400 + \$1200 = \$1600.

51. Let x = the amount invested at 5%. Then
 $2x - 400$ = the amount invested at 6.5%.
Use $I = prt$ with $t = 1$. Make a table.

Principal	Rate (as a decimal)	Interest
x	0.05	$0.05x$
$2x - 400$	0.065	$0.065(2x - 400)$
	Total →	298

The last column gives the equation.

$$\begin{array}{rcl} \text{Interest} & + & \text{interest} & = & \text{total} \\ \text{at 5\%} & & \text{at 6.5\%} & & \text{interest.} \\ 0.05x & + & 0.065(2x - 400) & = & 298 \end{array}$$

$$50x + 65(2x - 400) = 298,000 \quad \text{Multiply by 1000.}$$

$$50x + 130x - 26,000 = 298,000$$

$$180x - 26,000 = 298,000$$

$$180x = 324,000$$

$$x = \frac{324,000}{180} = 1800$$

She invested \$1800 at 5% and

$$2x - 400 = 2(1800) - 400 = \$3200 \text{ at 6.5\%}.$$

Check \$1800 @ 5% = \$90 and
\$3200 @ 6.5% = \$208; \$90 + \$208 = \$298.

52. Let x = the amount invested at 4.5%. Then
 $4x + 1000$ = the amount invested at 6%.
Use $I = prt$ with $t = 1$. Make a table.

Principal	Rate (as a decimal)	Interest
x	0.045	$0.045x$
$4x + 1000$	0.06	$0.06(4x + 1000)$
	Total →	801

The last column gives the equation.

$$\begin{array}{rcl} \text{Interest} & + & \text{interest} & = & \text{total} \\ \text{at 4.5\%} & & \text{at 6\%} & & \text{interest.} \\ 0.045x & + & 0.06(4x + 1000) & = & 801 \end{array}$$

$$45x + 60(4x + 1000) = 801,000 \quad \text{Multiply by 1000.}$$

$$45x + 240x + 60,000 = 801,000$$

$$285x + 60,000 = 801,000$$

$$285x = 741,000$$

$$x = \frac{741,000}{285} = 2600$$

He invested \$2600 at 4.5% and

$$4x + 1000 = 4(2600) + 1000 = \$11,400 \text{ at 6\%}.$$

Check \$2600 @ 4.5% = \$117 and
\$11,400 @ 6% = \$684; \$117 + \$684 = \$801.

53. Let x = the amount of additional money to be
invested at 4%.
Use $I = prt$ with $t = 1$. Make a table.
Use the fact that the total return on the two
investments is 6%.

Principal	Rate (as a decimal)	Interest
27,000	0.07	$0.07(27,000)$
x	0.04	$0.04x$
$27,000 + x$	0.06	$0.06(27,000 + x)$

The last column gives the equation.

$$\begin{array}{rcl} \text{Interest} & + & \text{interest} & = & \text{interest} \\ \text{at 7\%} & & \text{at 4\%} & & \text{at 6\%} \\ 0.07(27,000) & + & 0.04x & = & 0.06(27,000 + x) \end{array}$$

$$7(27,000) + 4x = 6(27,000 + x) \quad \text{Multiply by 100.}$$

$$189,000 + 4x = 162,000 + 6x$$

$$27,000 = 2x$$

$$13,500 = x$$

continued

They should invest \$13,500 at 4%.

Check \$27,000 @ 7% = \$1890 and
\$13,500 @ 4% = \$540;
\$1890 + \$540 = \$2430, which is the same as
(\$27,000 + \$13,500) @ 6%.

54. Let x = the amount of additional money to be invested at 5%.

Use $I = prt$ with $t = 1$. Make a table.

Use the fact that the total return on the two investments is 6%.

Principal	Rate (as a decimal)	Interest
17,000	0.065	$0.065(17,000)$
x	0.05	$0.05x$
$17,000 + x$	0.06	$0.06(17,000 + x)$

The last column gives the equation.

$$\begin{array}{rcl} \text{Interest} & + & \text{interest} & = & \text{interest} \\ \text{at 6.5\%} & & \text{at 5\%} & & \text{at 6\%} \\ 0.065(17,000) & + & 0.05x & = & 0.06(17,000 + x) \end{array}$$

$$65(17,000) + 50x = 60(17,000 + x)$$

Multiply by 1000.

$$\begin{aligned} 1,105,000 + 50x &= 1,020,000 + 60x \\ 85,000 &= 10x \\ 8500 &= x \end{aligned}$$

She should invest \$8500 at 6%.

Check \$17,000 @ 6.5% = \$1105 and
\$8500 @ 5% = \$425;
\$1105 + \$425 = \$1530, which is the same as
(\$17,000 + \$8500) @ 6%.

55. Let x = the number of liters of 10% acid solution needed. Make a table.

Liters of Solution	Percent (as a decimal)	Liters of Pure Acid
10	0.04	$0.04(10) = 0.4$
x	0.10	$0.10x$
$x + 10$	0.06	$0.06(x + 10)$

$$\begin{array}{rcl} \text{Acid in 4\%} & + & \text{acid in 10\%} & = & \text{acid in 6\%} \\ \text{solution} & & \text{solution} & & \text{solution.} \\ 0.4 & + & 0.10x & = & 0.06(x + 10) \end{array}$$

$$\begin{aligned} 0.4 + 0.10x &= 0.06x + 0.6 \\ \text{Distributive property} \\ 0.04x &= 0.2 \\ \text{Subtract } 0.06x \text{ and } 0.4. \\ x &= 5 \quad \text{Divide by } 0.04. \end{aligned}$$

Five liters of the 10% solution are needed.

Check 4% of 10 is 0.4 and 10% of 5 is 0.5;
 $0.4 + 0.5 = 0.9$, which is the same as 6% of
(10 + 5).

56. Let x = the number of liters of 14% alcohol solution needed. Make a chart.

Liters of Solution	Percent (as a decimal)	Liters of Pure Alcohol
x	0.14	$0.14x$
20	0.50	$0.50(20) = 10$
$x + 20$	0.30	$0.30(x + 20)$

$$\begin{array}{rcl} \text{Alcohol in} & + & \text{alcohol in} & = & \text{alcohol in} \\ 14\% \text{ solution} & & 50\% \text{ solution} & & 30\% \text{ solution.} \\ 0.14x & + & 10 & = & 0.30(x + 20) \end{array}$$

$$\begin{aligned} 14x + 1000 &= 30(x + 20) && \text{Multiply by 100.} \\ 14x + 1000 &= 30x + 600 \\ -16x &= -400 && \text{Subtract } 30x \text{ and } 1000. \\ x &= 25 && \text{Divide by } -16. \end{aligned}$$

25 L of 14% solution must be added.

Check 14% of 25 is 3.5 and 50% of 20 is 10;
 $3.5 + 10 = 13.5$, which is the same as 30% of
(25 + 20).

57. Let x = the number of liters of the 20% alcohol solution. Make a table.

Liters of Solution	Percent (as a decimal)	Liters of Pure Alcohol
12	0.12	$0.12(12) = 1.44$
x	0.20	$0.20x$
$x + 12$	0.14	$0.14(x + 12)$

$$\begin{array}{rcl} \text{Alcohol in} & + & \text{alcohol in} & = & \text{alcohol in} \\ 12\% \text{ solution} & & 20\% \text{ solution} & & 14\% \text{ solution.} \\ 1.44 & + & 0.20x & = & 0.14(x + 12) \end{array}$$

$$\begin{aligned} 144 + 20x &= 14(x + 12) && \text{Multiply by 100.} \\ 144 + 20x &= 14x + 168 && \text{Distributive property} \\ 6x &= 24 && \text{Subtract } 14x \text{ and } 144. \\ x &= 4 && \text{Divide by 6.} \end{aligned}$$

4L of 20% alcohol solution are needed.

Check 12% of 12 is 1.44 and 20% of 4 is 0.8;
 $1.44 + 0.8 = 2.24$, which is the same as 14% of
(12 + 4).

58. Let x = the number of liters of 10% alcohol solution. Make a chart.

Liters of Solution	Percent (as a decimal)	Liters of Pure Alcohol
x	0.10	$0.10x$
40	0.50	$0.50(40) = 20$
$x + 40$	0.40	$0.40(x + 40)$

$$\begin{array}{rcl} \text{Alcohol in} & + & \text{alcohol in} & = & \text{alcohol in} \\ 10\% \text{ solution} & & 50\% \text{ solution} & & 40\% \text{ solution.} \\ 0.10x & + & 20 & = & 0.40(x + 40) \end{array}$$

$$\begin{aligned}
 1x + 200 &= 4(x + 40) && \text{Multiply by 10.} \\
 x + 200 &= 4x + 160 \\
 -3x &= -40 && \text{Subtract } 4x \text{ and } 200. \\
 x &= \frac{40}{3} \text{ or } 13\frac{1}{3} && \text{Divide by } -3.
 \end{aligned}$$

$13\frac{1}{3}$ L of 10% solution should be added.

Check 50% of 40 is 20 and 10% of $\frac{40}{3}$ is $\frac{4}{3}$; $20 + \frac{4}{3} = 21\frac{1}{3}$, which is the same as 40% of $(\frac{40}{3} + 40)$.

59. Let x = the amount of pure dye used (pure dye is 100% dye). Make a table.

Gallons of Solution	Percent (as a decimal)	Gallons of Pure Dye
x	1	$1x = x$
4	0.25	$0.25(4) = 1$
$x + 4$	0.40	$0.40(x + 4)$

Write the equation from the last column in the table.

$$\begin{aligned}
 x + 1 &= 0.4(x + 4) \\
 x + 1 &= 0.4x + 1.6 && \text{Distributive property} \\
 0.6x &= 0.6 && \text{Subtract } 0.4x \text{ and } 1. \\
 x &= 1 && \text{Divide by } 0.6.
 \end{aligned}$$

One gallon of pure (100%) dye is needed.

Check 100% of 1 is 1 and 25% of 4 is 1; $1 + 1 = 2$, which is the same as 40% of $(1 + 4)$.

60. Let x = the number of gallons of water. Make a chart.

Gallons of Solution	Percent (as a decimal)	Gallons of Pure Insecticide
x	0	$0(x) = 0$
6	0.04	$0.04(6) = 0.24$
$x + 6$	0.03	$0.03(x + 6)$

$$\begin{array}{rcl}
 \text{Insecticide} & + & \text{insecticide} & = & \text{insecticide} \\
 \text{in water} & & \text{in 4\% solution} & & \text{in 3\% solution.} \\
 0 & + & 0.24 & = & 0.03(x + 6)
 \end{array}$$

$$\begin{aligned}
 0 + 24 &= 3(x + 6) && \text{Multiply by 100.} \\
 24 &= 3x + 18 && \text{Distributive property} \\
 6 &= 3x && \text{Subtract 18.} \\
 2 &= x && \text{Divide by 3.}
 \end{aligned}$$

2 gallons of water should be added.

Check 4% of 6 is 0.24, which is the same as 3% of $(2 + 6)$.

61. Let x = the amount of \$6 per lb nuts. Make a table.

Pounds of nuts	Cost per lb	Total Cost
50	\$2	$2(50) = 100$
x	\$6	$6x$
$x + 50$	\$5	$5(x + 50)$

The total value of the \$2 per lb nuts and the \$6 per lb nuts must equal the value of the \$5 per lb nuts.

$$\begin{aligned}
 100 + 6x &= 5(x + 50) \\
 100 + 6x &= 5x + 250 \\
 x &= 150
 \end{aligned}$$

He should use 150 lb of \$6 nuts.

Check 50 pounds of the \$2 per lb nuts are worth \$100 and 150 pounds of the \$6 per lb nuts are worth \$900; $\$100 + \$900 = \$1000$, which is the same as $(50 + 150)$ pounds worth \$5 per lb.

62. Let x = the number of ounces of 2¢ per oz tea. Make a table.

Ounces of Tea	Cost per oz	Total Cost
x	2¢ or 0.02	$0.02x$
100	5¢ or 0.05	$0.05(100) = 5$
$x + 100$	3¢ or 0.03	$0.03(x + 100)$

$$\begin{array}{rcl}
 \text{Cost of} & + & \text{cost of} & = & \text{cost of} \\
 2¢ \text{ tea} & & 5¢ \text{ tea} & & 3¢ \text{ tea.} \\
 0.02x & + & 5 & = & 0.03(x + 100)
 \end{array}$$

$$\begin{aligned}
 2x + 500 &= 3(x + 100) && \text{Multiply by 100.} \\
 2x + 500 &= 3x + 300 && \text{Distributive property} \\
 200 &= x && \text{Subtract } 2x \text{ and } 300.
 \end{aligned}$$

200 oz of 2¢ per oz tea should be used.

Check 200 oz of 2¢ per oz tea is worth \$4 and 100 oz of 5¢ per oz tea is worth \$5; $\$4 + \$5 = \$9$, which is the same value as $(200 + 100)$ oz of 3¢ per oz tea.

63. We cannot expect the final mixture to be worth more than each of the ingredients. Answers will vary.
64. Let x = the number of liters of 30% acid solution. Make a chart.

Liters of Solution	Percent (as a decimal)	Liters of Pure Acid
x	0.30	$0.30x$
15	0.50	$0.50(15) = 7.5$
$x + 15$	0.60	$0.60(x + 15)$

$$\begin{array}{rcl}
 \text{Acid in 30\%} & + & \text{acid in 50\%} & = & \text{acid in 60\%} \\
 \text{solution} & & \text{solution} & & \text{solution.} \\
 0.30x & + & 7.5 & = & 0.60(x + 15)
 \end{array}$$

continued

$$\begin{aligned}
 3x + 75 &= 6x + 90 && \text{Multiply by 10.} \\
 -3x &= 15 && \text{Subtract } 6x \text{ and } 75. \\
 x &= -5 && \text{Divide by } -3.
 \end{aligned}$$

The solution, -5 , is impossible since the number of liters of 30% acid solution cannot be negative. Therefore, this problem has no solution.

65. (a) Let x = the amount invested at 5%.
 $800 - x$ = the amount invested at 10%.
- (b) Let y = the amount of 5% acid used.
 $800 - y$ = the amount of 10% acid used.
66. Organize the information in a table.

(a)

Principal	Percent (as a decimal)	Interest
x	0.05	$0.05x$
$800 - x$	0.10	$0.10(800 - x)$
800	0.0875	$0.0875(800)$

The amount of interest earned at 5% and 10% is found in the last column of the table, $0.05x$ and $0.10(800 - x)$.

(b)

Liters of Solution	Percent (as a decimal)	Liters of Pure Acid
y	0.05	$0.05y$
$800 - y$	0.10	$0.10(800 - y)$
800	0.0875	$0.0875(800)$

The amount of pure acid in the 5% and 10% mixtures is found in the last column of the table, $0.05y$ and $0.10(800 - y)$.

67. Refer to the tables for Exercise 66. In each case, the last column gives the equation.
- (a) $0.05x + 0.10(800 - x) = 0.0875(800)$
- (b) $0.05y + 0.10(800 - y) = 0.0875(800)$
68. In both cases, multiply by 10,000 to clear the decimals.
- (a)
- $$\begin{aligned}
 0.05x + 0.10(800 - x) &= 0.0875(800) \\
 500x + 1000(800 - x) &= 875(800) \\
 500x + 800,000 - 1000x &= 700,000 \\
 -500x &= -100,000 \\
 x &= 200
 \end{aligned}$$

Jack invested \$200 at 5% and
 $800 - x = 800 - 200 = \$600$ at 10%.

- (b)
- $$\begin{aligned}
 0.05y + 0.10(800 - y) &= 0.0875(800) \\
 500y + 1000(800 - y) &= 875(800) \\
 500y + 800,000 - 1000y &= 700,000 \\
 -500y &= -100,000 \\
 y &= 200
 \end{aligned}$$

Jill used 200 L of 5% acid solution and
 $800 - y = 800 - 200 = 600$ L of 10% acid solution.

69. The processes used to solve Problems A and B were virtually the same. Aside from the variables chosen, the problem information was organized in similar tables and the equations solved were the same.

70. $d = rt$; $r = 50$, $t = 4$

$$d = 50(4) = 200$$

71. $P = 2L + 2W$; $L = 10$, $W = 6$

$$P = 2(10) + 2(6) = 20 + 12 = 32$$

72. $P = 2L + 2W$; $P = 80$, $L = 34$

$$80 = 2(34) + 2W$$

$$80 = 68 + 2W$$

$$12 = 2W$$

$$6 = W$$

73. $P = a + b + c$; $b = 13$, $c = 14$, $P = 46$

$$46 = a + 13 + 14$$

$$46 = a + 27$$

$$19 = a$$

74. $A = \frac{1}{2}h(b + B)$; $A = 156$, $b = 12$, $B = 14$

$$156 = \frac{1}{2}h(12 + 14)$$

$$312 = h(26)$$

$$h = \frac{312}{26} = 12$$

75. $d = rt$; $d = 75$, $t = 15$

$$75 = r(15)$$

$$r = \frac{75}{15} = 5$$

2.4 Further Applications of Linear Equations

2.4 Classroom Examples

1. Let x = the number of dimes.
 Then $26 - x$ = the number of half-dollars.

Number of Coins	Denomination	Value
x	0.10	$0.10x$
$26 - x$	0.50	$0.50(26 - x)$
	Total →	8.60

Multiply the number of coins by the denominations, and add the results to get 8.60.

$$0.10x + 0.50(26 - x) = 8.60$$

$$1x + 5(26 - x) = 86 \quad \text{Multiply by 10.}$$

$$1x + 130 - 5x = 86$$

$$-4x = -44$$

$$x = 11$$

He has 11 dimes and $26 - 11 = 15$ half-dollars.

2. Let x = the amount of time needed for the cars to be 420 mi apart.

Make a table. Use the formula $d = rt$, that is, find each distance by multiplying rate by time.

	Rate	Time	Distance
Northbound Car	60	x	$60x$
Southbound Car	45	x	$45x$
Total			420

The total distance traveled is the sum of the distances traveled by each car, since they are traveling in opposite directions. This total is 420 mi.

$$\begin{aligned} 60x + 45x &= 420 \\ 105x &= 420 \\ x &= \frac{420}{105} = 4 \end{aligned}$$

The cars will be 420 mi apart in 4 hr.

3. Let x = the driving speed.
Then $x - 12$ = the bus speed.

Make a table. Use the formula $d = rt$, that is, find each distance by multiplying rate by time.

	Rate	Time	Distance
Car	x	$\frac{1}{2}$	$\frac{1}{2}x$
Bus	$x - 12$	$\frac{3}{4}$	$\frac{3}{4}(x - 12)$

The distances are equal.

$$\begin{aligned} \frac{1}{2}x &= \frac{3}{4}(x - 12) \\ 2x &= 3(x - 12) \quad \text{Multiply by 4.} \\ 2x &= 3x - 36 \\ 36 &= x \end{aligned}$$

The distance he travels to work is

$$\frac{1}{2}x = \frac{1}{2}(36) = 18 \text{ miles.}$$

4. The sum of the three measures must equal 180° .

$$\begin{aligned} x + (x + 61) + (2x + 7) &= 180 \\ 4x + 68 &= 180 \\ 4x &= 112 \\ x &= 28 \end{aligned}$$

The angles measure 28° , $28 + 61 = 89^\circ$, and $2(28) + 7 = 63^\circ$.

2.4 Section Exercises

1. The total amount is

$$\begin{aligned} 38(0.05) + 26(0.10) &= 1.90 + 2.60 \\ &= \$4.50. \end{aligned}$$

2. Use $d = rt$, or $t = \frac{d}{r}$.

Substitute 7700 for d and 480 for r .

$$t = \frac{7700}{480} \approx 16.04$$

Its travel time is approximately 16 hours.

3. Use $d = rt$, or $r = \frac{d}{t}$. Substitute 300 for d and 5 for t .

$$r = \frac{300}{5} = 60$$

His rate was 60 mph.

4. Use $P = 4s$ or $s = \frac{P}{4}$.

Substitute 40 for P .

$$s = \frac{40}{4} = 10$$

The length of each side of the square is 10 in. This is also the length of each side of the equilateral triangle. To find the perimeter of the equilateral triangle, use $P = 3s$.

Substitute 10 for s .

$$P = 3(10) = 30$$

The perimeter would be 30 inches.

5. The problem asks for the distance Jeff travels to the workplace, so we must multiply the rate, 10 mph, by the time, $\frac{3}{4}$ hr, to get the distance, 7.5 mi.
6. Begin by subtracting 36° from 180° . Then find half of this difference to get the measure of each of the equal angles.
7. No, the answers must be whole numbers because they represent the number of coins.
8. Since the distance d is in miles and the rate r is x miles per hour, we must have the time t in hours. 30 minutes = $\frac{30}{60} = \frac{1}{2}$ hour, so

$$d = rt = x\left(\frac{1}{2}\right) \text{ or } \frac{1}{2}x.$$

9. Let x = the number of pennies. Then x is also the number of dimes, and $44 - 2x$ is the number of quarters.

Number of Coins	Denomination	Value
x	0.01	$0.01x$
x	0.10	$0.10x$
$44 - 2x$	0.25	$0.25(44 - 2x)$
44	← Totals →	4.37

The sum of the values must equal the total value.

$$\begin{aligned} 0.01x + 0.10x + 0.25(44 - 2x) &= 4.37 \\ x + 10x + 25(44 - 2x) &= 437 \\ &\text{Multiply by 100.} \end{aligned}$$

$$\begin{aligned} x + 10x + 1100 - 50x &= 437 \\ -39x + 1100 &= 437 \\ -39x &= -663 \\ x &= 17 \end{aligned}$$

continued

There are 17 pennies, 17 dimes, and
 $44 - 2(17) = 10$ quarters.

Check The number of coins is
 $17 + 17 + 10 = 44$ and the value of the coins is
 $\$.01(17) + \$.10(17) + \$.25(10) = \$4.37$,
 as required.

10. Let x = the number of nickels and the number of quarters. Then $2x$ is the number of half-dollars.

Number of Coins	Denomination	Value
x	0.05	$0.05x$
x	0.25	$0.25x$
$2x$	0.50	$0.50(2x)$
	Total →	2.60

The sum of the values must equal the total value.

$$\begin{aligned}
 0.05x + 0.25x + 0.50(2x) &= 2.60 \\
 5x + 25x + 50(2x) &= 260 \\
 \text{Multiply by 100.} \\
 5x + 25x + 100x &= 260 \\
 130x &= 260 \\
 x &= 2
 \end{aligned}$$

She found 2 nickels, 2 quarters, and $2(2) = 4$ half-dollars.

Check The value of the coins is
 $\$.05(2) + \$.25(2) + \$.50(4) = \$2.60$, as required.

11. Let x = the number of loonies. Then $37 - x$ is the number of toonies.

Number of Coins	Denomination	Value
x	1	$1x$
$37 - x$	2	$2(37 - x)$
37	← Totals →	51

The sum of the values must equal the total value.

$$\begin{aligned}
 1x + 2(37 - x) &= 51 \\
 x + 74 - 2x &= 51 \\
 -x + 74 &= 51 \\
 23 &= x
 \end{aligned}$$

She has 23 loonies and $37 - 23 = 14$ toonies.

Check The value of the coins is
 $\$1(23) + \$2(14) = \$51$, as required.

12. Let x = the number of \$1 bills. Then $119 - x$ is the number of \$5 bills.

Number of Bills	Denomination	Value
x	1	$1x$
$119 - x$	5	$5(119 - x)$
119	← Totals →	347

The sum of the values must equal the total value.

$$\begin{aligned}
 1x + 5(119 - x) &= 347 \\
 x + 595 - 5x &= 347 \\
 -4x &= -248 \\
 x &= 62
 \end{aligned}$$

He has 62 \$1 bills and $119 - 62 = 57$ \$5 bills.

Check The value of the bills is
 $\$1(62) + \$5(57) = \$62 + \$285 = \$347$,
 as required.

13. Let x = the number of \$10 coins. Then $41 - x$ is the number of \$20 coins.

Number of Coins	Denomination	Value
x	10	$10x$
$41 - x$	20	$20(41 - x)$
41	← Totals →	540

The sum of the values must equal the total value.

$$\begin{aligned}
 10x + 20(41 - x) &= 540 \\
 10x + 820 - 20x &= 540 \\
 -10x &= -280 \\
 x &= 28
 \end{aligned}$$

He has 28 \$10 coins and $41 - 28 = 13$ \$20 coins.

Check The value of the coins is
 $\$10(28) + \$20(13) = \$540$, as required.

14. Let x = the number of two-cent pieces. Then $3x$ is the number of three-cent pieces.

Number of Coins	Denomination	Value
x	0.02	$0.02x$
$3x$	0.03	$0.03(3x)$
	Total →	2.42

The sum of the values must equal the total value.

$$\begin{aligned}
 0.02x + 0.03(3x) &= 2.42 \\
 2x + 3(3x) &= 242 \\
 \text{Multiply both sides by 100.} \\
 2x + 9x &= 242 \\
 11x &= 242 \\
 x &= 22
 \end{aligned}$$

She has 22 two-cent pieces and $3(22) = 66$ three-cent pieces.

Check The value of the coins is
 $\$0.02(22) + \$0.03(66) = \$2.42$, as required.

15. Let x = the number of adult tickets sold. Then $2010 - x$ = the number of senior tickets sold.

Cost of Ticket	Number Sold	Amount Collected
\$12	x	$12x$
\$7	$2010 - x$	$7(2010 - x)$
Totals	2010	\$18,430

Write the equation from the last column of the table.

$$12x + 7(2010 - x) = 18,430$$

$$12x + 14,070 - 7x = 18,430$$

$$5x = 4360$$

$$x = 872$$

872 adult tickets were sold; $2010 - 872 = 1138$ senior tickets were sold.

Check The amount collected was
 $\$12(872) + \$7(1138)$
 $= \$10,464 + \$7966 = \$18,430$, as required.

16. Let x = the number of student tickets sold. Then $480 - x$ = the number of nonstudent tickets sold.

Cost of Ticket	Number Sold	Amount Collected
\$5	x	$5x$
\$8	$480 - x$	$8(480 - x)$
Totals	480	\$2895

Write the equation from the last column of the table.

$$5x + 8(480 - x) = 2895$$

$$5x + 3840 - 8x = 2895$$

$$-3x = -945$$

$$x = 315$$

315 student tickets were sold; $480 - 315 = 165$ nonstudent tickets were sold.

Check The amount collected was
 $\$5(315) + \$8(165) = \$1575 + \$1320 = \$2895$, as required.

17. $d = rt$, so

$$r = \frac{d}{t} = \frac{100}{12.37} \approx 8.08$$

Her rate was about 8.08 m/sec.

18. $d = rt$, so

$$r = \frac{d}{t} = \frac{400}{52.82} \approx 7.57$$

Her rate was about 7.57 m/sec.

19. $d = rt$, so

$$r = \frac{d}{t} = \frac{400}{47.63} \approx 8.40$$

His rate was about 8.40 m/sec.

20. $d = rt$, so

$$r = \frac{d}{t} = \frac{400}{44.00} \approx 9.09$$

His rate was about 9.09 m/sec.

21. Let t = the time until they are 110 mi apart. Use the formula $d = rt$. Complete the table.

	Rate	Time	Distance
First Steamer	22	t	$22t$
Second Steamer	22	t	$22t$
			110

The total distance traveled is the sum of the distances traveled by each steamer, since they are traveling in opposite directions. This total is 110 mi.

$$22t + 22t = 110$$

$$44t = 110$$

$$t = \frac{110}{44} = \frac{5}{2} \text{ or } 2\frac{1}{2}$$

It will take them $2\frac{1}{2}$ hr.

Check Each steamer traveled $22(2.5) = 55$ miles for a total of $2(55) = 110$ miles, as required.

22. Let t = the time it takes for the trains to be 315 km apart.

Use the formula $d = rt$. Complete the table.

	Rate	Time	Distance
First Train	85	t	$85t$
Second Train	95	t	$95t$
			315

The total distance traveled is the sum of the distances traveled by each train, since they are traveling in opposite directions. This total is 315 km.

$$85t + 95t = 315$$

$$180t = 315$$

$$t = \frac{315}{180} = \frac{7}{4} \text{ or } 1\frac{3}{4}$$

It will take the trains $1\frac{3}{4}$ hr before they are 315 km apart.

Check The first train traveled $85(1.75) = 148.75$ km and the second train traveled $95(1.75) = 166.25$ km. The sum is 315 km, as required.

23. Let t = Mulder's time. Then
 $t - \frac{1}{2}$ = Scully's time.

	Rate	Time	Distance
Mulder	65	t	$65t$
Scully	68	$t - \frac{1}{2}$	$68(t - \frac{1}{2})$

The distances are equal.

$$\begin{aligned} 65t &= 68(t - \frac{1}{2}) \\ 65t &= 68t - 34 \\ -3t &= -34 \\ t &= \frac{34}{3} \text{ or } 11\frac{1}{3} \end{aligned}$$

Mulder's time will be $11\frac{1}{3}$ hr. Since he left at 8:30 A.M., $11\frac{1}{3}$ hr or 11 hr 20 min later is 7:50 P.M.

Check Mulder's distance was $65(\frac{34}{3}) = 736\frac{2}{3}$ miles. Scully's distance was $68(\frac{34}{3} - \frac{1}{2}) = 68(\frac{65}{6}) = 736\frac{2}{3}$, as required.

24. Let x = Lois' travel time.
 Since Clark leaves 15 minutes after Lois, and $\frac{15}{60} = \frac{1}{4}$ hr, $x - \frac{1}{4}$ = time for Clark.
 Complete the table using the formula $rt = d$.

	Rate	Time	Distance
Lois	35	x	$35x$
Clark	40	$x - \frac{1}{4}$	$40(x - \frac{1}{4})$

Since Lois and Clark are going in opposite directions, we add their distances to get 140 mi.

$$\begin{aligned} 35x + 40(x - \frac{1}{4}) &= 140 \\ 35x + 40x - 10 &= 140 \\ 75x &= 150 \\ x &= 2 \end{aligned}$$

They will be 140 mi apart at
 8 A.M. + 2 hr = 10 A.M.

Check Lois' distance was $35(2) = 70$. Clark's distance was $40(2 - \frac{1}{4}) = 40(\frac{7}{4}) = 70$. The sum is 140 miles, as required.

25. Let x = her average speed on Sunday. Then
 $x + 5$ = her average speed on Saturday.

	Rate	Time	Distance
Saturday	$x + 5$	3.6	$3.6(x + 5)$
Sunday	x	4	$4x$

The distances are equal.

$$\begin{aligned} 3.6(x + 5) &= 4x \\ 3.6x + 18 &= 4x \\ 18 &= 0.4x && \text{Subtract } 3.6x. \\ x &= \frac{18}{0.4} = 45 \end{aligned}$$

Her average speed on Sunday was 45 mph.

Check On Sunday, 4 hours @ 45 mph = 180 miles. On Saturday, 3.6 hours @ 50 mph = 180 miles. The distances are equal.

26. Let x = her biking speed. Then
 $x - 7$ = her walking speed.

	Rate	Time	Distance
Walking	$x - 7$	$\frac{40}{60} = \frac{2}{3}$ hr	$\frac{2}{3}(x - 7)$
Biking	x	$\frac{12}{60} = \frac{1}{5}$ hr	$\frac{1}{5}x$

The distances are equal.

$$\begin{aligned} \frac{2}{3}(x - 7) &= \frac{1}{5}x \\ 10(x - 7) &= 3x && \text{Multiply by 15.} \\ 10x - 70 &= 3x \\ 7x &= 70 \\ x &= 10 \end{aligned}$$

The distance from her house to the train station is $\frac{1}{5}x = \frac{1}{5}(10) = 2$ miles.

Check The distance walking is $(3 \text{ mph})(\frac{2}{3} \text{ hr}) = 2$ miles. The distance biking is $(10 \text{ mph})(\frac{1}{5} \text{ hr}) = 2$ miles. The distances are equal.

27. Let x = Anne's time. Then
 $x + \frac{1}{2}$ = Johnny's time.

	Rate	Time	Distance
Anne	60	x	$60x$
Johnny	50	$x + \frac{1}{2}$	$50(x + \frac{1}{2})$

The total distance is 80.

$$\begin{aligned} 60x + 50(x + \frac{1}{2}) &= 80 \\ 60x + 50x + 25 &= 80 \\ 110x &= 55 \\ x &= \frac{55}{110} = \frac{1}{2} \end{aligned}$$

They will meet $\frac{1}{2}$ hr after Anne leaves.

Check Anne travels $60(\frac{1}{2}) = 30$ miles. Johnny travels $50(\frac{1}{2} + \frac{1}{2}) = 50$ miles. The sum of the distances is 80 miles, as required.

28. Let x = her rate (speed) during the first part of the trip. Then $x - 25$ = her rate during rush hour traffic. Make a table using the formula $rt = d$.

	Rate	Time	Distance
First Part	x	2	$2x$
Rush Hour	$x - 25$	$\frac{1}{2}$	$\frac{1}{2}(x - 25)$

The total distance was 125 miles.

$$\begin{aligned}
 2x + \frac{1}{2}(x - 25) &= 125 \\
 4x + x - 25 &= 250 && \text{Multiply by 2.} \\
 5x &= 275 \\
 x &= 55
 \end{aligned}$$

The speed during the first part of the trip was 55 mph.

Check The distance traveled during the first part of the trip was $55(2) = 110$ miles. The distance traveled during the second part of the trip was $(55 - 25)(0.5) = 15$ miles. The sum of the distances is 125 miles, as required.

29. The sum of the measures of the three angles of a triangle is 180° .

$$\begin{aligned}
 (x - 30) + (2x - 120) + \left(\frac{1}{2}x + 15\right) &= 180 \\
 \frac{7}{2}x - 135 &= 180 \\
 7x - 270 &= 360 \\
 &&& \text{Multiply by 2.} \\
 7x &= 630 \\
 x &= 90
 \end{aligned}$$

With $x = 90$, the three angle measures become

$$\begin{aligned}
 (90 - 30)^\circ &= 60^\circ, \\
 [2(90) - 120]^\circ &= 60^\circ, \\
 \text{and } \left[\frac{1}{2}(90) + 15\right]^\circ &= 60^\circ.
 \end{aligned}$$

30. The sum of the measures of the three angles of a triangle is 180° .

$$\begin{aligned}
 (x + 15) + (10x - 20) + (x + 5) &= 180 \\
 12x &= 180 \\
 x &= 15
 \end{aligned}$$

With $x = 15$, the three angle measures become

$$\begin{aligned}
 (15 + 15)^\circ &= 30^\circ, \\
 (10 \cdot 15 - 20)^\circ &= 130^\circ, \\
 \text{and } (15 + 5)^\circ &= 20^\circ.
 \end{aligned}$$

31. The sum of the measures of the three angles of a triangle is 180° .

$$\begin{aligned}
 (3x + 7) + (9x - 4) + (4x + 1) &= 180 \\
 16x + 4 &= 180 \\
 16x &= 176 \\
 x &= 11
 \end{aligned}$$

With $x = 11$, the three angle measures become

$$\begin{aligned}
 (3 \cdot 11 + 7)^\circ &= 40^\circ, \\
 (9 \cdot 11 - 4)^\circ &= 95^\circ, \\
 \text{and } (4 \cdot 11 + 1)^\circ &= 45^\circ.
 \end{aligned}$$

32. The sum of the measures of the three angles of a triangle is 180° .

$$\begin{aligned}
 (2x + 7) + (x + 61) + x &= 180 \\
 4x + 68 &= 180 \\
 4x &= 112 \\
 x &= 28
 \end{aligned}$$

With $x = 28$, the three angle measures become

$$\begin{aligned}
 (2 \cdot 28 + 7)^\circ &= 63^\circ, \\
 (28 + 61)^\circ &= 89^\circ, \text{ and } 28^\circ.
 \end{aligned}$$

33. The sum of the measures of the angles of a triangle is 180° .

$$\begin{aligned}
 x + 2x + 60 &= 180 \\
 3x + 60 &= 180 \\
 3x &= 120 \\
 x &= 40
 \end{aligned}$$

The measures of the unknown angles are 40° and $2x = 80^\circ$.

34. Two angles which form a straight line add to 180° , so $180^\circ - 60^\circ = 120^\circ$. The measure of the unknown angle is 120° .

35. The sum of the measures of the unknown angles in Exercise 33 is $40^\circ + 80^\circ = 120^\circ$. This is equal to the measure of the angle in Exercise 34.

36. The sum of the measures of angles ① and ② is equal to the measure of angle ③.

37. Vertical angles have equal measure.

$$\begin{aligned}
 8x + 2 &= 7x + 17 \\
 x &= 15
 \end{aligned}$$

$$8 \cdot 15 + 2 = 122 \text{ and } 7 \cdot 15 + 17 = 122.$$

The angles are both 122° .

38. Vertical angles have equal measure.

$$\begin{aligned}
 9 - 5x &= 25 - 3x \\
 9 &= 25 + 2x \\
 -16 &= 2x \\
 -8 &= x
 \end{aligned}$$

$$9 - 5(-8) = 49 \text{ and } 25 - 3(-8) = 49.$$

The angles are both 49° .

39. The sum of the two angles is 90° .

$$\begin{aligned}
 (5x - 1) + 2x &= 90 \\
 7x - 1 &= 90 \\
 7x &= 91 \\
 x &= 13
 \end{aligned}$$

The measures of the two angles are $[5(13) - 1]^\circ = 64^\circ$ and $[2(13)]^\circ = 26^\circ$.

40. Supplementary angles have an angle measure sum of 180° .

$$\begin{aligned}(3x + 5) + (5x + 15) &= 180 \\ 8x + 20 &= 180 \\ 8x &= 160 \\ x &= 20\end{aligned}$$

With $x = 20$, the two angle measures become

$$\begin{aligned}(3 \cdot 20 + 5)^\circ &= 65^\circ \\ \text{and } (5 \cdot 20 + 15)^\circ &= 115^\circ.\end{aligned}$$

41. Let x = the first consecutive integer. Then $x + 1$ will be the second consecutive integer, and $x + 2$ will be the third consecutive integer.

The sum of the first and twice the second is 17 more than twice the third.

$$\begin{aligned}x + 2(x + 1) &= 2(x + 2) + 17 \\ x + 2x + 2 &= 2x + 4 + 17 \\ 3x + 2 &= 2x + 21 \\ x &= 19\end{aligned}$$

Since $x = 19$, $x + 1 = 20$, and $x + 2 = 21$.
The three consecutive integers are 19, 20, and 21.

42. Let x = the first integer. Then $x + 1$, $x + 2$, and $x + 3$ are the next three consecutive integers.
The sum of the first three integers is 54 more than the fourth.

$$\begin{aligned}x + (x + 1) + (x + 2) &= (x + 3) + 54 \\ 3x + 3 &= x + 57 \\ 2x &= 54 \\ x &= 27\end{aligned}$$

The four consecutive integers are 27, 28, 29, and 30.

43. Let x = the current age. Then $x + 1$ will be the age next year.
The sum of these ages will be 103 years.

$$\begin{aligned}x + (x + 1) &= 103 \\ 2x + 1 &= 103 \\ 2x &= 102 \\ x &= 51\end{aligned}$$

If my current age is 51, in 10 years I will be

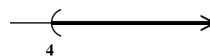
$$51 + 10 = 61 \text{ years old.}$$

44. Let x = the page number on one page. Then $x + 1$ is the page number on the next page.
The sum of the page numbers is 193.

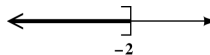
$$\begin{aligned}x + (x + 1) &= 193 \\ 2x + 1 &= 193 \\ 2x &= 192 \\ x &= 96\end{aligned}$$

The page numbers are 96 and 97.

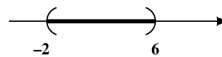
45. $(4, \infty)$ is equivalent to $x > 4$.



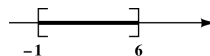
46. $(-\infty, -2]$ is equivalent to $x \leq -2$.



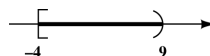
47. $(-2, 6)$ is equivalent to $-2 < x < 6$.



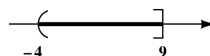
48. $[-1, 6]$ is equivalent to $-1 \leq x \leq 6$.



49. $[-4, 9)$ is equivalent to $-4 \leq x < 9$.



50. $(-4, 9]$ is equivalent to $-4 < x \leq 9$.



Summary Exercises on Solving Applied Problems

1. Let x = the width of the rectangle. Then $x + 3$ is the length of the rectangle.

If the length were decreased by 2 inches and the width were increased by 1 inch, the perimeter would be 24 inches. Use the formula $P = 2L + 2W$, and substitute 24 for P , $(x + 3) - 2$ or $x + 1$ for L , and $x + 1$ for W .

$$\begin{aligned}P &= 2L + 2W \\ 24 &= 2(x + 1) + 2(x + 1) \\ 24 &= 2x + 2 + 2x + 2 \\ 24 &= 4x + 4 \\ 20 &= 4x \\ 5 &= x\end{aligned}$$

The width of the rectangle is 5 inches, and the length is $5 + 3 = 8$ inches.

2. Let x = the width. Then $2x$ is the length (twice the width).
Use $P = 2L + 2W$ and add one more width to cut the area into two parts. Thus, an equation is

$$\begin{aligned}2L + 2W + W &= 210. \\ 2(2x) + 2x + x &= 210 \\ 7x &= 210 \\ x &= 30\end{aligned}$$

The width is 30 meters, and the length is $2(30) = 60$ meters.

3. Let x = the regular price of the item.
The sale price after a 37% (or 0.37) discount was \$32.09, so an equation is

$$\begin{aligned}x - 0.37x &= 32.09. \\0.63x &= 32.09 \\x &= \frac{32.09}{0.63} \approx 50.94\end{aligned}$$

To the nearest cent, the regular price was \$50.94.

4. Let x = the regular price of the DVD recorder.
The sale price after a discount of 40% (or 0.40) was \$255, so an equation is

$$\begin{aligned}x - 0.40x &= 255. \\0.60x &= 255 \\x &= 425\end{aligned}$$

The regular price of the DVD recorder was \$425.

5. Let x = the amount invested at 4%.
Then $2x$ is the amount invested at 5%.
Use $I = prt$ with $t = 1$ yr. Make a table.

Principal	Rate (as a decimal)	Interest
x	0.04	$0.04x$
$2x$	0.05	$0.05(2x) = 0.10x$
	Total →	112

The last column gives the equation.

$$\begin{array}{rclcl} \text{Interest} & + & \text{interest} & = & \text{total} \\ \text{at 4\%} & & \text{at 5\%} & & \text{interest.} \\ 0.04x & + & 0.10x & = & 112 \end{array}$$

$$\begin{aligned}4x + 10x &= 11,200 && \text{Multiply by 100.} \\14x &= 11,200 \\x &= 800\end{aligned}$$

\$800 is invested at 4% and $2(\$800) = \$1600$ at 5%.

Check $\$800 @ 4\% = \$32$ and
 $\$1600 @ 5\% = \$80$; $\$32 + \$80 = \$112$

6. Let x = the amount invested at 3%.
Then $x + 2000$ is the amount invested at 4%.
Use $I = prt$ with $t = 1$ yr. Make a table.

Principal	Rate (as a decimal)	Interest
x	0.03	$0.03x$
$x + 2000$	0.04	$0.04(x + 2000)$
	Total →	920

The last column gives the equation.

$$\begin{array}{rclcl} \text{Interest} & + & \text{interest} & = & \text{total} \\ \text{at 3\%} & & \text{at 4\%} & & \text{interest.} \\ 0.03x & + & 0.04(x + 2000) & = & 920 \end{array}$$

$$\begin{aligned}3x + 4(x + 2000) &= 92,000 && \text{Multiply by 100.} \\3x + 4x + 8000 &= 92,000 \\7x &= 84,000 \\x &= 12,000\end{aligned}$$

\$12,000 is invested at 3% and \$14,000 is invested at 4%.

Check $\$12,000 @ 3\% = \$360$ and
 $\$14,000 @ 4\% = \$560$; $\$360 + \$560 = \$920$

7. Let x = the number of Emmy awards won by *The Simpsons*. Then $2x - 9$ = the number of Emmys won by *Frasier*. The total number of Emmys won by both was 51.

$$\begin{aligned}x + (2x - 9) &= 51 \\3x - 9 &= 51 \\3x &= 60 \\x &= 20\end{aligned}$$

The Simpsons won 20 Emmys and *Frasier* won $2(20) - 9 = 31$ Emmys.

8. Let x = the amount grossed by *Star Wars: Episode IV—A New Hope*. Then $x + 139.8$ = the amount grossed by *Titanic* (in millions). Together they grossed \$1061.8 million.

$$\begin{aligned}x + (x + 139.8) &= 1061.8 \\2x + 139.8 &= 1061.8 \\2x &= 922 \\x &= 461\end{aligned}$$

Star Wars: Episode IV—A New Hope grossed \$461 million and *Titanic* grossed $461 + 139.8 = \$600.8$ million.

9. Let t = the time it will take until John and Pat meet. Use $d = rt$ and make a table.

	Rate	Time	Distance
John	60	t	$60t$
Pat	28	t	$28t$

The total distance is 440 miles.

$$\begin{aligned}60t + 28t &= 440 \\88t &= 440 \\t &= 5\end{aligned}$$

It will take 5 hours for John and Pat to meet.

Check John traveled $60(5) = 300$ miles and Pat traveled $28(5) = 140$ miles; $300 + 140 = 440$, as required.

10. Let x = Negussie's rate. Then
 $x - 1.1$ = Ndereba's rate.

	Rate	Time	Distance
Negussie	x	2.196	$2.196x$
Ndereba	$x - 1.1$	2.420	$2.420(x - 1.1)$

The distances are equal.

$$\begin{aligned}
 2.196x &= 2.420(x - 1.1) \\
 2196x &= 2420(x - 1.1) && \text{Multiply by 1000.} \\
 2196x &= 2420x - 2662 \\
 2662 &= 224x \\
 x &= \frac{2662}{224} \approx 11.88
 \end{aligned}$$

Negussie's rate was 11.88 mph and Ndereba's rate was 10.78 mph.

Check Negussie's distance was $(11.88 \text{ mph})(2.196 \text{ hr}) \approx 26.09$ miles. Ndereba's distance was $(10.78 \text{ mph})(2.420 \text{ hr}) \approx 26.09$ miles. (The values are slightly different due to rounding.)

11. Let x = the number of liters of the 5% drug solution.

Liters of Solution	Percent (as a decimal)	Liters of Pure Drug
20	0.10	$20(0.10) = 2$
x	0.05	$0.05x$
$20 + x$	0.08	$0.08(20 + x)$

$$\begin{array}{rcl}
 \text{Drug} & + & \text{drug} & = & \text{drug} \\
 \text{in 10\%} & & \text{in 5\%} & & \text{in 8\%} \\
 2 & + & 0.05x & = & 0.08(20 + x)
 \end{array}$$

$$\begin{aligned}
 200 + 5x &= 8(20 + x) && \text{Multiply by 100.} \\
 200 + 5x &= 160 + 8x \\
 40 &= 3x \\
 x &= \frac{40}{3} \text{ or } 13\frac{1}{3}
 \end{aligned}$$

The pharmacist should add $13\frac{1}{3}$ L.

Check 10% of 20 is 2 and 5% of $\frac{40}{3}$ is $\frac{2}{3}$; $2 + \frac{2}{3} = \frac{8}{3}$, which is the same as 8% of $(20 + \frac{40}{3})$.

12. Let x = the number of kilograms of the metal that is 20% tin.

Kilograms of Metal	Percent Tin (as a decimal)	Kilograms of Pure Tin
80	0.70	$80(0.70) = 56$
x	0.20	$0.20x$
$80 + x$	0.50	$0.50(80 + x)$

$$\begin{array}{rcl}
 \text{Tin} & + & \text{tin} & = & \text{tin} \\
 \text{in 70\%} & & \text{in 20\%} & & \text{in 50\%} \\
 56 & + & 0.20x & = & 0.50(80 + x)
 \end{array}$$

$$\begin{aligned}
 560 + 2x &= 5(80 + x) && \text{Multiply by 10.} \\
 560 + 2x &= 400 + 5x \\
 160 &= 3x \\
 x &= \frac{160}{3} \text{ or } 53\frac{1}{3}
 \end{aligned}$$

$53\frac{1}{3}$ kilograms should be added.

Check 70% of 80 is 56 and 20% of $\frac{160}{3}$ is $\frac{32}{3}$; $56 + \frac{32}{3} = 66\frac{2}{3}$, which is the same as 50% of $(80 + \frac{160}{3})$.

13. Let x = the number of \$5 bills. Then $126 - x$ is the number of \$10 bills.

Number of Bills	Denomination	Value
x	5	$5x$
$126 - x$	10	$10(126 - x)$
126	← Totals →	840

The sum of the values must equal the total value.

$$\begin{aligned}
 5x + 10(126 - x) &= 840 \\
 5x + 1260 - 10x &= 840 \\
 -5x &= -420 \\
 x &= 84
 \end{aligned}$$

There are 84 \$5 bills and $126 - 84 = 42$ \$10 bills.

Check The value of the bills is $\$5(84) + \$10(42) = \$840$, as required.

14. Let x = the number of \$6 tickets sold. Then $2460 - x$ = the number of \$8 tickets sold.

Number Sold	Cost of Ticket	Amount Collected
x	\$6	$6x$
$2460 - x$	\$8	$8(2460 - x)$
2460	← Totals →	\$18,060

Write the equation from the last column of the table.

$$\begin{aligned}
 6x + 8(2460 - x) &= 18,060 \\
 6x + 19,680 - 8x &= 18,060 \\
 -2x &= -1620 \\
 x &= 810
 \end{aligned}$$

810 \$6 tickets were sold and $2460 - 810 = 1650$ \$8 tickets were sold.

Check The amount collected was $\$6(810) + \$8(1650) = \$4860 + \$13,200 = \$18,060$, as required.

15. The sum of the measures of the three angles of a triangle is 180° .

$$\begin{aligned}x + (6x - 50) + (x - 10) &= 180 \\8x - 60 &= 180 \\8x &= 240 \\x &= 30\end{aligned}$$

With $x = 30$, the three angle measures become

$$\begin{aligned}(6 \cdot 30 - 50)^\circ &= 130^\circ, \\(30 - 10)^\circ &= 20^\circ, \text{ and } 30^\circ.\end{aligned}$$

16. In the figure, the two angles are supplementary, so their sum is 180° .

$$\begin{aligned}(10x + 7) + (7x + 3) &= 180 \\17x + 10 &= 180 \\17x &= 170 \\x &= 10\end{aligned}$$

The two angle measures are $10(10) + 7 = 107^\circ$ and $7(10) + 3 = 73^\circ$.

17. Let x = the least integer. Then $x + 1$ is the middle integer and $x + 2$ is the greatest integer.

"The sum of the least and greatest of three consecutive integers is 32 more than the middle integer" translates to

$$\begin{aligned}x + (x + 2) &= 32 + (x + 1). \\2x + 2 &= x + 33 \\x &= 31\end{aligned}$$

The three consecutive integers are 31, 32, and 33.

Check The sum of the least and greatest integers is $31 + 33 = 64$, which is the same as 32 more than the middle integer.

18. Let x = the first odd integer. Then $x + 2$ is the next odd integer.

"If the lesser of two consecutive odd integers is doubled, the result is 7 more than the greater of the two integers" translates to

$$\begin{aligned}2(x) &= 7 + (x + 2). \\2x &= x + 9 \\x &= 9\end{aligned}$$

The two consecutive odd integers are 9 and 11.

Check Doubling the lesser gives us $2(9) = 18$, which is equal to 7 more than 11.

19. Let x = the length of the shortest side. Then $2x$ is the length of the middle side and $3x - 2$ is the length of the longest side.

The perimeter is 34 inches. Using $P = a + b + c$ gives us

$$\begin{aligned}x + 2x + (3x - 2) &= 34. \\6x - 2 &= 34 \\6x &= 36 \\x &= 6\end{aligned}$$

The lengths of the three sides are 6 inches, $2(6) = 12$ inches, and $3(6) - 2 = 16$ inches.

Check The sum of the lengths of the three sides is $6 + 12 + 16 = 34$ inches, as required.

20. Let x = the length of the rectangle. Then the perimeter is $43 + x$.

$$\begin{aligned}P &= 2L + 2W \\43 + x &= 2x + 2(10) \quad \text{Let } W = 10. \\43 + x &= 2x + 20 \\23 &= x\end{aligned}$$

The length of the rectangle is 23 inches.

Check $P = 2L + 2W = 2(23) + 2(10) = 66$ inches, which is 43 inches more than the length.

2.5 Linear Inequalities in One Variable

2.5 Classroom Examples

1. $k - 5 > 1$
 $k > 6$ Add 5.

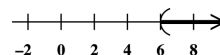
Check Substitute 6 for k in $k - 5 = 1$.

$$\begin{aligned}6 - 5 &= 1 \quad ? \\1 &= 1 \quad \text{True}\end{aligned}$$

This shows that 6 is the boundary point. Choose 0 and 7 as test points.

$k - 5 > 1$	
$\text{Let } k = 0.$ $0 - 5 > 1 \quad ?$ $-5 > 1 \quad \text{False}$ 0 is not in the solution set.	$\text{Let } k = 7.$ $7 - 5 > 1 \quad ?$ $2 > 1 \quad \text{True}$ 7 is in the solution set.

The check confirms that $(6, \infty)$ is the solution set.



$$\begin{aligned}
 2. \quad 5x + 3 &\geq 4x - 1 \\
 x + 3 &\geq -1 && \text{Subtract } 4x. \\
 x &\geq -4 && \text{Subtract } -3.
 \end{aligned}$$

Check Substitute -4 for x in $5x + 3 = 4x - 1$.

$$\begin{aligned}
 5(-4) + 3 &= 4(-4) - 1 \quad ? \\
 -17 &= -17 && \text{True}
 \end{aligned}$$

So -4 satisfies the equality part of $\geq$. Choose -5 and 0 as test points.

$$5x + 3 \geq 4x - 1$$

Let $x = -5$.

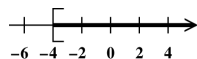
$$\begin{aligned}
 5(-5) + 3 &\geq 4(-5) - 1 \quad ? \\
 -22 &\geq -21 && \text{False} \\
 -5 &\text{ is not in the solution set.}
 \end{aligned}$$

Let $x = 0$.

$$\begin{aligned}
 5(0) + 3 &\geq 4(0) - 1 \quad ? \\
 3 &\geq -1 && \text{True}
 \end{aligned}$$

0 is in the solution set.

The check confirms that $[-4, \infty)$ is the solution set.



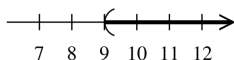
$$\begin{aligned}
 3. \quad (a) \quad 4m &\leq -100 \\
 \frac{4m}{4} &\leq \frac{-100}{4} && \text{Divide by } 4 > 0; \text{ do not} \\
 m &\leq -25 && \text{reverse the symbol.}
 \end{aligned}$$

Check that the solution set is the interval $(-\infty, -25]$.



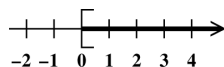
$$\begin{aligned}
 (b) \quad -9m &< -81 \\
 \frac{-9m}{-9} &> \frac{-81}{-9} && \text{Divide by } -9 < 0; \\
 m &> 9 && \text{reverse the symbol.}
 \end{aligned}$$

Check that the solution set is the interval $(9, \infty)$.



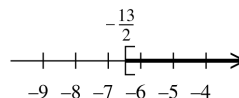
$$\begin{aligned}
 4. \quad 6(x - 1) + 3x &\geq -x - 3(x + 2) \\
 6x - 6 + 3x &\geq -x - 3x - 6 \\
 9x - 6 &\geq -4x - 6 \\
 13x - 6 &\geq -6 \\
 13x &\geq 0 \\
 \frac{13x}{13} &\geq \frac{0}{13} \\
 x &\geq 0
 \end{aligned}$$

Check that the solution set is the interval $[0, \infty)$.



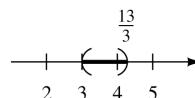
$$\begin{aligned}
 5. \quad \frac{1}{4}(m + 3) + 2 &\leq \frac{3}{4}(m + 8) \\
 4\left[\frac{1}{4}(m + 3) + 2\right] &\leq 4\left[\frac{3}{4}(m + 8)\right] \\
 (m + 3) + 8 &\leq 3(m + 8) && \text{Multiply by } 4. \\
 m + 11 &\leq 3m + 24 \\
 -2m + 11 &\leq 24 && \text{Subtract } 3m. \\
 -2m &\leq 13 && \text{Subtract } 11. \\
 m &\geq -\frac{13}{2} && \text{Divide by } -2; \\
 &&& \text{reverse symbol.}
 \end{aligned}$$

Check that the solution set is the interval $[-\frac{13}{2}, \infty)$.



$$\begin{aligned}
 6. \quad 5 &< 3x - 4 < 9 \\
 9 &< 3x < 13 && \text{Add } 4 \text{ to each part.} \\
 \frac{9}{3} &< \frac{3x}{3} < \frac{13}{3} && \text{Divide each part by } 3. \\
 3 &< x < \frac{13}{3}
 \end{aligned}$$

Check that the solution set is the interval $(3, \frac{13}{3})$.



7. **Step 2**
Let h = the number of hours she can rent the leaf blower.

Step 3

She must pay \$5, plus $\$1.75h$, to rent the leaf blower for h hours, and this amount must be *no more than* \$26.

Cost of renting	is no more than	26 dollars.
$\downarrow$	$\downarrow$	$\downarrow$
$5 + 1.75h$	$\leq$	26

Step 4

$$\begin{aligned}
 1.75h &\leq 21 && \text{Subtract } 5. \\
 h &\leq 12 && \text{Divide by } 1.75.
 \end{aligned}$$

Step 5

She can use the leaf blower for a maximum of 12 hr. (She may use it for less time, as indicated by the inequality $h \leq 12$.)

Step 6

If Marge uses the leaf blower for 12 hr, she will spend $5 + 1.75(12) = 26$, the maximum amount.

8. Let x = the grade he must make on the fourth test.
To find the average of the four tests, add them and divide by 4. This average must be at least 90, that is, greater than or equal to 90.

$$\begin{aligned}\frac{92 + 90 + 84 + x}{4} &\geq 90 \\ \frac{266 + x}{4} &\geq 90 \\ 266 + x &\geq 360 && \text{Multiply by 4.} \\ x &\geq 94 && \text{Subtract 266.}\end{aligned}$$

Michael must score at least 94 on the fourth test.

Check $\frac{92 + 90 + 84 + 94}{4} = \frac{360}{4} = 90$

A score of 94 or more will give an average of at least 90, as required.

2.5 Section Exercises

1. $x \leq 3$

In interval notation, this inequality is written $(-\infty, 3]$. The bracket indicates that 3 is included. The answer is choice **D**.

2. $x > 3$

In interval notation, this inequality is written $(3, \infty)$. The parenthesis indicates that 3 is not included. The answer is choice **C**.

3. $x < 3$

In interval notation, this inequality is written $(-\infty, 3)$. The parenthesis indicates that 3 is not included. The graph of this inequality is shown in choice **B**.

4. $x \geq 3$

In interval notation, this inequality is written $[3, \infty)$. The bracket indicates that 3 is included. The graph of this inequality is shown in choice **A**.

5. $-3 \leq x \leq 3$

In interval notation, this inequality is written $[-3, 3]$. The brackets indicate that -3 and 3 are included. The answer is choice **F**.

6. $-3 < x < 3$

In interval notation, this inequality is written $(-3, 3)$. The parentheses indicate that neither -3 nor 3 is included. The answer is choice **E**.

7. (a) The wind speed s of a Category 4 hurricane is between 131 mph and 155 mph [inclusive], which can be described by the three-part inequality $131 \leq s \leq 155$.

(b) The wind speed s of a Category 5 hurricane is greater than 155 mph, which can be described by the inequality $s > 155$.

(c) The storm surge x of a Category 3 hurricane is between 9 ft and 12 ft [inclusive], which can be described by the three-part inequality $9 \leq x \leq 12$.

(d) The storm surge x of a Category 5 hurricane is greater than 18 ft, which can be described by the inequality $x > 18$.

8. (a) A *small frame* corresponds to a wrist circumference x that is less than 6.75 inches, which can be described by the inequality $x < 6.75$.

(b) A *medium frame* corresponds to a wrist circumference x that is between 6.75 and 7.25 inches [inclusive], which can be described by the three-part inequality $6.75 \leq x \leq 7.25$.

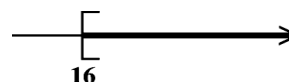
(c) A *large frame* corresponds to a wrist circumference x that is greater than 7.25 inches, which can be described by the inequality $x > 7.25$.

9. Since $4 > 0$, the student should not have reversed the direction of the inequality symbol when dividing by 4. We reverse the inequality symbol only when multiplying or dividing by a *negative* number. The solution set is $[-16, \infty)$.

10. If an endpoint is an element of the solution set, then this is shown on the graph using a bracket. If an endpoint is *not* an element of the solution set, then this is shown on the graph using a parenthesis.

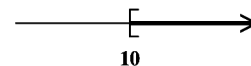
11. $x - 4 \geq 12$
 $x \geq 16$ Add 4.

Check that the solution set is the interval $[16, \infty)$.



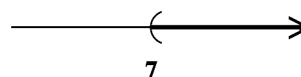
12. $t - 3 \geq 7$
 $t \geq 10$ Add 3.

Check that the solution set is the interval $[10, \infty)$.



13. $3k + 1 > 22$
 $3k > 21$ Subtract 1.
 $k > 7$ Divide by 3.

Check that the solution set is the interval $(7, \infty)$.

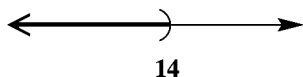


14. $5z + 6 < 76$

$5z < 70$ Subtract 6.

$z < 14$ Divide by 5.

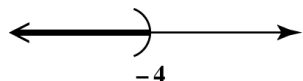
Check that the solution set is the interval $(-\infty, 14)$.



15. $4x < -16$

$x < -4$ Divide by 4.

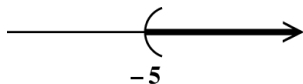
Check that the solution set is the interval $(-\infty, -4)$.



16. $2m > -10$

$m > -5$ Divide by 2.

Check that the solution set is the interval $(-5, \infty)$.



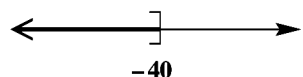
17. $-\frac{3}{4}r \geq 30$

Multiply both sides by $-\frac{4}{3}$, and reverse the inequality symbol.

$-\frac{4}{3}(-\frac{3}{4}r) \leq -\frac{4}{3}(30)$

$r \leq -40$

Check that the solution set is the interval $(-\infty, -40]$.



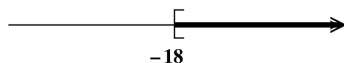
18. $-\frac{2}{3}x \leq 12$

Multiply each side by $-\frac{3}{2}$ and reverse the inequality symbol.

$-\frac{3}{2}(-\frac{2}{3}x) \geq -\frac{3}{2}(12)$

$x \geq -18$

Check that the solution set is the interval $[-18, \infty)$.



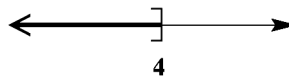
19. $-1.3m \geq -5.2$

Divide both sides by -1.3 , and reverse the inequality symbol.

$\frac{-1.3m}{-1.3} \leq \frac{-5.2}{-1.3}$

$m \leq 4$

Check that the solution set is the interval $(-\infty, 4]$.



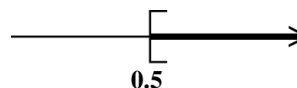
20. $-2.5y \leq -1.25$

Divide by -2.5 and reverse the inequality symbol.

$$\frac{-2.5y}{-2.5} \geq \frac{-1.25}{-2.5}$$

$$y \geq 0.5$$

Check that the solution set is the interval $[0.5, \infty)$.

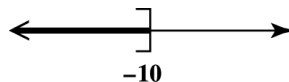


21. $5t + 2 \leq -48$

$5t \leq -50$ Subtract 2.

$t \leq -10$ Divide by 5.

Check that the solution set is the interval $(-\infty, -10]$.

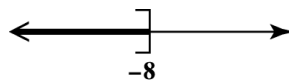


22. $4x + 1 \leq -31$

$4x \leq -32$ Subtract 1.

$x \leq -8$ Divide by 4.

Check that the solution set is the interval $(-\infty, -8]$.



23. $\frac{5z - 6}{8} < 8$

$8\left(\frac{5z - 6}{8}\right) < 8 \cdot 8$ Multiply by 8.

$5z - 6 < 64$

$5z < 70$ Add 6.

$z < 14$ Divide by 5.

Check Let $z = 14$ in the equation $\frac{5z - 6}{8} = 8$.

$\frac{5(14) - 6}{8} = 8 \quad ?$

$\frac{64}{8} = 8 \quad ?$

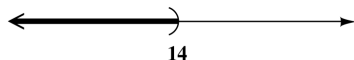
$8 = 8$ True

This shows that 14 is the boundary point. Now test a number on each side of 14. We choose 0 and 20.

$\frac{5z - 6}{8} < 8$

Let $z = 0$.	Let $z = 20$.
$\frac{5(0) - 6}{8} < 8 ?$	$\frac{5(20) - 6}{8} < 8 ?$
$-\frac{6}{8} < 8$ True	$\frac{94}{8}$ (or $11\frac{6}{8}$) < 8 False
0 is in the solution set.	20 is not in the solution set.

The check confirms that $(-\infty, 14)$ is the solution set.



24. $\frac{3k - 1}{4} > 5$

$$4\left(\frac{3k - 1}{4}\right) > 4(5) \quad \text{Multiply by 4.}$$

$$3k - 1 > 20$$

$$3k > 21 \quad \text{Add 1.}$$

$$k > 7 \quad \text{Divide by 3.}$$

Check Let $k = 7$ in the equation $\frac{3k - 1}{4} = 5$.

$$\frac{3(7) - 1}{4} = 5 ?$$

$$\frac{20}{4} = 5 ?$$

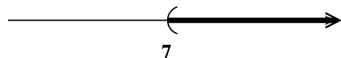
$$5 = 5 \quad \text{True}$$

This shows that 7 is the boundary point. Now test a number on each side of 7. We choose 0 and 10.

$$\frac{3k - 1}{4} > 5$$

Let $k = 0$.	Let $k = 10$.
$\frac{3(0) - 1}{4} > 5 ?$	$\frac{3(10) - 1}{4} > 5 ?$
$-\frac{1}{4} > 5$ False	$\frac{29}{4} > 5$ True
0 is not in the solution set.	10 is in the solution set.

The check confirms that $(7, \infty)$ is the solution set.



25. $\frac{2k - 5}{-4} > 5$

Multiply both sides by -4 , and reverse the inequality symbol.

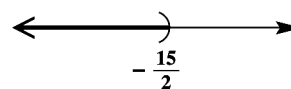
$$-4\left(\frac{2k - 5}{-4}\right) < -4(5)$$

$$2k - 5 < -20$$

$$2k < -15 \quad \text{Add 5.}$$

$$k < -\frac{15}{2} \quad \text{Divide by 2.}$$

Check that the solution set is the interval $(-\infty, -\frac{15}{2})$.



26. $\frac{3z - 2}{-5} < 6$

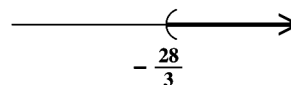
Multiply by -5 and reverse the inequality symbol.

$$3z - 2 > -30$$

$$3z > -28 \quad \text{Add 2.}$$

$$z > -\frac{28}{3} \quad \text{Divide by 3.}$$

Check that the solution set is the interval $(-\frac{28}{3}, \infty)$.



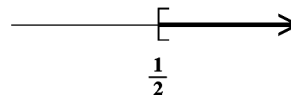
27. $6x - 4 \geq -2x$

$$8x - 4 \geq 0 \quad \text{Add 2x.}$$

$$8x \geq 4 \quad \text{Add 4.}$$

$$x \geq \frac{4}{8} = \frac{1}{2} \quad \text{Divide by 8.}$$

Check that the solution set is the interval $[\frac{1}{2}, \infty)$.

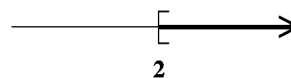


28. $-2m + 8 \leq 2m$

$$8 \leq 4m \quad \text{Add 2m.}$$

$$2 \leq m \quad \text{or} \quad m \geq 2$$

Check that the solution set is the interval $[2, \infty)$.



29. $m - 2(m - 4) \leq 3m$

$$m - 2m + 8 \leq 3m$$

$$-m + 8 \leq 3m$$

$$8 \leq 4m \quad \text{Add m.}$$

$$2 \leq m \quad \text{or} \quad m \geq 2$$

Check that the solution set is the interval $[2, \infty)$.



30. $x + 4(2x - 1) \geq x$

$$x + 8x - 4 \geq x$$

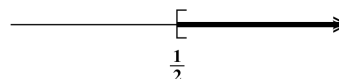
$$9x - 4 \geq x$$

$$9x \geq x + 4 \quad \text{Add 4.}$$

$$8x \geq 4 \quad \text{Subtract x.}$$

$$x \geq \frac{1}{2} \quad \text{Divide by 8.}$$

Check that the solution set is the interval $[\frac{1}{2}, \infty)$.



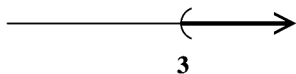
31. $-(4 + r) + 2 - 3r < -14$

$$\begin{aligned} -4 - r + 2 - 3r &< -14 && \text{Distributive property} \\ -4r - 2 &< -14 && \text{Combine terms.} \end{aligned}$$

$$-4r < -12 \quad \text{Add 2.}$$

Divide by -4 , and reverse the inequality symbol.

$$r > 3$$

Check that the solution set is the interval $(3, \infty)$.

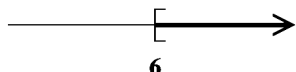
32. $-(9 + k) - 5 + 4k \geq 4$

$$-9 - k - 5 + 4k \geq 4$$

$$-14 + 3k \geq 4$$

$$3k \geq 18$$

$$k \geq 6$$

Check that the solution set is the interval $[6, \infty)$.

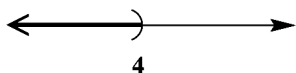
33. $-3(z - 6) > 2z - 2$

$$-3z + 18 > 2z - 2 \quad \text{Distributive property}$$

$$-5z > -20 \quad \text{Subtract } 2z \text{ and } 18.$$

Divide by -5 , and reverse the inequality symbol.

$$z < 4$$

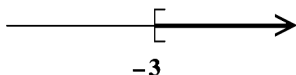
Check that the solution set is the interval $(-\infty, 4)$.

34. $-2(x + 4) \leq 6x + 16$

$$-2x - 8 \leq 6x + 16$$

$$-8x \leq 24$$

$$x \geq -3 \quad \text{Reverse symbol.}$$

Check that the solution set is the interval $[-3, \infty)$.

35. $\frac{2}{3}(3k - 1) \geq \frac{3}{2}(2k - 3)$

Multiply both sides by 6 to clear the fractions.

$$6 \cdot \frac{2}{3}(3k - 1) \geq 6 \cdot \frac{3}{2}(2k - 3)$$

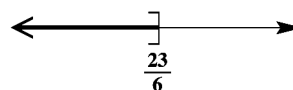
$$4(3k - 1) \geq 9(2k - 3)$$

$$12k - 4 \geq 18k - 27 \quad \text{Distributive property}$$

$$-6k \geq -23 \quad \text{Subtract } 18k; \text{ add 4.}$$

Divide by -6 , and reverse the inequality symbol.

$$k \leq \frac{23}{6}$$

Check that the solution set is the interval $(-\infty, \frac{23}{6}]$.

36. $\frac{7}{5}(10m - 1) < \frac{2}{3}(6m + 5)$

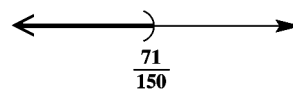
$$15 \cdot \frac{7}{5}(10m - 1) < 15 \cdot \frac{2}{3}(6m + 5)$$

$$21(10m - 1) < 10(6m + 5)$$

$$210m - 21 < 60m + 50$$

$$150m < 71$$

$$m < \frac{71}{150}$$

Check that the solution set is the interval $(-\infty, \frac{71}{150})$.

37. $-\frac{1}{4}(p + 6) + \frac{3}{2}(2p - 5) < 10$

Multiply each term by 4 to clear the fractions.

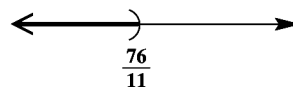
$$-1(p + 6) + 6(2p - 5) < 40$$

$$-p - 6 + 12p - 30 < 40$$

$$11p - 36 < 40$$

$$11p < 76$$

$$p < \frac{76}{11}$$

Check that the solution set is the interval $(-\infty, \frac{76}{11})$.

38. $\frac{3}{5}(k - 2) - \frac{1}{4}(2k - 7) \leq 3$

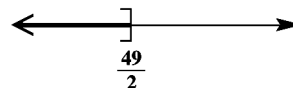
$$20 \cdot \frac{3}{5}(k - 2) - 20 \cdot \frac{1}{4}(2k - 7) \leq 20 \cdot 3$$

$$12(k - 2) - 5(2k - 7) \leq 60$$

$$12k - 24 - 10k + 35 \leq 60$$

$$2k \leq 49$$

$$k \leq \frac{49}{2}$$

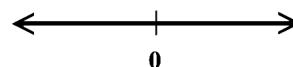
Check that the solution set is the interval $(-\infty, \frac{49}{2}]$.

39. $3(2x - 4) - 4x < 2x + 3$

$$6x - 12 - 4x < 2x + 3$$

$$2x - 12 < 2x + 3$$

$$-12 < 3 \quad \text{True}$$

The statement is true for all values of x . Therefore, the original inequality is true for any real number.Check that the solution set is the interval $(-\infty, \infty)$.

40. $7(4 - x) + 5x < 2(16 - x)$

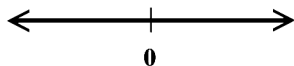
$28 - 7x + 5x < 32 - 2x$

$28 - 2x < 32 - 2x$

$28 < 32$ True

This statement is true for all values of x .

Check that the solution set is the interval $(-\infty, \infty)$.



41. $8(\frac{1}{2}x + 3) < 8(\frac{1}{2}x - 1)$

$4x + 24 < 4x - 8$

$24 < -8$ False

This is a false statement, so the inequality is a contradiction.

Check that the solution set is $\emptyset$.

42. $10x + 2(x - 4) < 12x - 10$

$10x + 2x - 8 < 12x - 10$

$12x - 8 < 12x - 10$

$-8 < -10$ False

This is a false statement, so the inequality is a contradiction.

Check that the solution set is $\emptyset$.

43. $5(x + 3) - 2(x - 4) = 2(x + 7)$

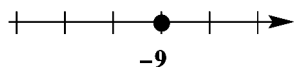
$5x + 15 - 2x + 8 = 2x + 14$

$3x + 23 = 2x + 14$

$x = -9$

Check that the solution set is $\{-9\}$.

The graph is the point -9 on a number line.



44. $5(x + 3) - 2(x - 4) > 2(x + 7)$

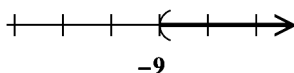
$5x + 15 - 2x + 8 > 2x + 14$

$3x + 23 > 2x + 14$

$x > -9$

Check that the solution set is the interval $(-9, \infty)$.

The graph extends from -9 to the right on a number line; -9 is not included in the graph.



45. $5(x + 3) - 2(x - 4) < 2(x + 7)$

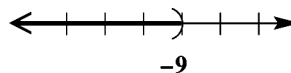
$5x + 15 - 2x + 8 < 2x + 14$

$3x + 23 < 2x + 14$

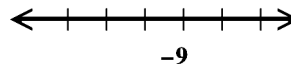
$x < -9$

Check that the solution set is the interval $(-\infty, -9)$.

The graph extends from -9 to the left on a number line; -9 is not included in the graph.



46. If we graph all the solution sets from Exercises 43–45; that is, $\{-9\}$, $(-9, \infty)$, and $(-\infty, -9)$, on the same number line, we will have graphed the set of all real numbers.



47. The solution set of the given equation is the point -3 on a number line. The solution set of the first inequality extends from -3 to the right (toward ∞) on the same number line. Based on Exercises 43–45, the solution set of the second inequality should then extend from -3 to the left (toward $-\infty$) on the number line. Complete the statement with $(-\infty, -3)$.

48. $-2 < x$ is the same as $x > -2$. This inequality represents all real numbers greater than -2 . Its graph is shown in choice A.

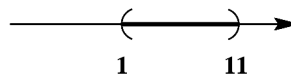
49. $-4 < x - 5 < 6$

Add 5 to each part of the inequality to isolate the variable x .

$-4 + 5 < x - 5 + 5 < 6 + 5$

$1 < x < 11$

Check that the solution set is the interval $(1, 11)$.



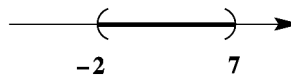
50. $-1 < x + 1 < 8$

Subtract 1 from each part to isolate the variable x .

$-1 - 1 < x + 1 - 1 < 8 - 1$

$-2 < x < 7$

Check that the solution set is the interval $(-2, 7)$.



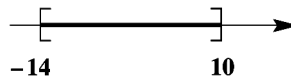
51. $-9 \leq k + 5 \leq 15$

Subtract 5 from each part.

$-9 - 5 \leq k + 5 - 5 \leq 15 - 5$

$-14 \leq k \leq 10$

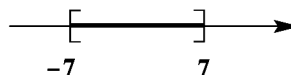
Check that the solution set is the interval $[-14, 10]$.



52. $-4 \leq m + 3 \leq 10$

$-7 \leq m \leq 7$ Subtract 3.

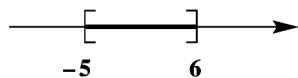
Check that the solution set is the interval $[-7, 7]$.



53. $-6 \leq 2z + 4 \leq 16$

$-10 \leq 2z \leq 12$ Subtract 4.

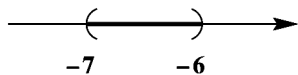
$-5 \leq z \leq 6$ Divide by 2.

Check that the solution set is the interval $[-5, 6]$.

54. $-15 < 3p + 6 < -12$

$-21 < 3p < -18$ Subtract 6.

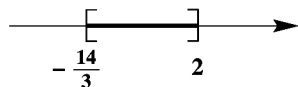
$-7 < p < -6$ Divide by 3.

Check that the solution set is the interval $(-7, -6)$.

55. $-19 \leq 3x - 5 \leq 1$

$-14 \leq 3x \leq 6$ Add 5.

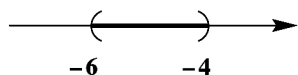
$-\frac{14}{3} \leq x \leq 2$ Divide by 3.

Check that the solution set is the interval $[-\frac{14}{3}, 2]$.

56. $-16 < 3t + 2 < -10$

$-18 < 3t < -12$ Subtract 2.

$-6 < t < -4$ Divide by 3.

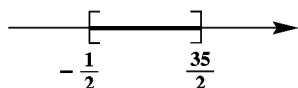
Check that the solution set is the interval $(-6, -4)$.

57. $-1 \leq \frac{2x-5}{6} \leq 5$

$-6 \leq 2x - 5 \leq 30$ Multiply by 6.

$-1 \leq 2x \leq 35$ Add 5.

$-\frac{1}{2} \leq x \leq \frac{35}{2}$ Divide by 2.

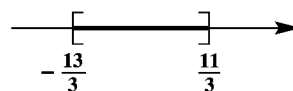
Check that the solution set is the interval $[-\frac{1}{2}, \frac{35}{2}]$.

58. $-3 \leq \frac{3m+1}{4} \leq 3$

$-12 \leq 3m + 1 \leq 12$ Multiply by 4.

$-13 \leq 3m \leq 11$ Subtract 1.

$-\frac{13}{3} \leq m \leq \frac{11}{3}$ Divide by 3.

Check that the solution set is the interval $[-\frac{13}{3}, \frac{11}{3}]$.

59. $4 \leq -9x + 5 < 8$

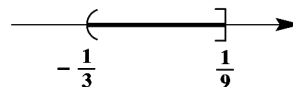
$-1 \leq -9x < 3$ Subtract 5.

Divide each part by -9 ; reverse the inequality symbols.

$\frac{1}{9} \geq x > -\frac{1}{3}$

The last inequality may be written as

$-\frac{1}{3} < x \leq \frac{1}{9}$.

Check that the solution set is the interval $(-\frac{1}{3}, \frac{1}{9}]$.

60. $4 \leq -2x + 3 < 8$

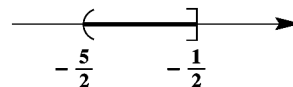
$1 \leq -2x < 5$ Subtract 3.

Divide each part by -2 and reverse the inequality symbols.

$-\frac{1}{2} \geq x > -\frac{5}{2}$

The last inequality may be written as

$-\frac{5}{2} < x \leq -\frac{1}{2}$.

Check that the solution set is the interval $(-\frac{5}{2}, -\frac{1}{2}]$.

61. Six times a number is between -12 and 12 .

$-12 < 6x < 12$

$-2 < x < 2$ Divide by 6.

This is the set of all numbers between -2 and 2 —that is, $(-2, 2)$.

62. Half a number is between -3 and 2 .

$-3 < \frac{1}{2}x < 2$

$-6 < x < 4$ Multiply by 2.

This is the set of all numbers between -6 and 4 —that is, $(-6, 4)$.

63. When 1 is added to twice a number, the result is greater than or equal to 7.

$2x + 1 \geq 7$

$2x \geq 6$ Subtract 1.

$x \geq 3$ Divide by 2.

This is the set of all numbers greater than or equal to 3—that is, $[3, \infty)$.

64. If 8 is subtracted from a number, then the result is at least 5.

$$\begin{aligned}x - 8 &\geq 5 \\x &\geq 13 \quad \text{Add 8.}\end{aligned}$$

This is the set of all numbers greater than or equal to 13—that is, $[13, \infty)$.

65. One third of a number is added to 6, giving a result of at least 3.

$$\begin{aligned}6 + \frac{1}{3}x &\geq 3 \\ \frac{1}{3}x &\geq -3 \quad \text{Subtract 6.} \\ x &\geq -9 \quad \text{Multiply by 3.}\end{aligned}$$

This is the set of all numbers greater than or equal to -9 —that is, $[-9, \infty)$.

66. Three times a number, minus 5, is no more than 7.

$$\begin{aligned}3x - 5 &\leq 7 \\ 3x &\leq 12 \quad \text{Add 5.} \\ x &\leq 4 \quad \text{Divide by 3.}\end{aligned}$$

This is the set of all numbers less than or equal to 4—that is, $(-\infty, 4]$.

67. Draw a horizontal line at the 90°F mark. It intersects the *upper* boundary of the forecasted highs in two places. The temperature is expected to be at least 90°F from about 2:30 P.M. to 6:00 P.M.
68. Draw a horizontal line at the 70°F mark. It intersects the *lower* boundary of the forecasted highs in two places. The temperature is predicted to be no more than 70°F from about 8:00 A.M. to 10:15 A.M. and after about 9:00 P.M.
69. Locate the dot at 5:39 P.M. and draw a vertical line through it. The line will intersect the lower and upper boundaries of the forecasted highs at about 84°F and 91°F , so the range of predicted temperatures is 84°F – 91°F .
70. Locate the dot at 8:45 A.M. and draw a vertical line through it. The line will intersect the lower and upper boundaries of the forecasted highs at about 65°F and 67°F , so the range of predicted temperatures is 65°F – 67°F .
71. Let x = his score on the third test. His average must be at least 84 (≥ 84). To find the average of three numbers, add them and divide by 3.

$$\begin{aligned}\frac{90 + 82 + x}{3} &\geq 84 \\ \frac{172 + x}{3} &\geq 84 \quad \text{Add.} \\ 172 + x &\geq 252 \quad \text{Multiply by 3.} \\ x &\geq 80 \quad \text{Subtract 172.}\end{aligned}$$

He must score at least 80 on his third test.

72. Let x = his score on the third test. His average must be at least 90.

$$\begin{aligned}\frac{92 + 96 + x}{3} &\geq 90 \\ 92 + 96 + x &\geq 270 \quad \text{Multiply by 3.} \\ 188 + x &\geq 270 \quad \text{Add.} \\ x &\geq 82 \quad \text{Subtract 188.}\end{aligned}$$

He must score at least 82 on his third test.

73. Let x = the number of months. The cost of Plan A is $54.99x$ and the cost of Plan B is $49.99x + 129$. To determine the number of months that would be needed to make Plan B less expensive, solve the following inequality.

$$\begin{aligned}\text{Plan B (cost)} &< \text{Plan A (cost)} \\ 49.99x + 129 &< 54.99x \\ 129 &< 5x \quad \text{Subtract } 49.99x. \\ 5x &> 129 \quad \text{Equivalent} \\ x &> \frac{129}{5} \quad [= 25.8] \quad \text{Divide by 5.}\end{aligned}$$

It will take 26 months for Plan B to be the better deal.

74. Let x = the number of miles driven. The cost of renting from Budget is $\$34.95$ plus the mileage cost of $\$0.25x$, while the cost of renting from U-Haul is $\$29.95$ plus the mileage cost of $\$0.28x$.

$$\begin{aligned}\text{Budget (cost)} &< \text{U-Haul (cost)} \\ 34.95 + 0.25x &< 29.95 + 0.28x \\ 5 + 0.25x &< 0.28x \\ 5 &< 0.03x \\ 0.03x &> 5 \\ x &> \frac{5}{0.03} \quad [= \frac{500}{3} \text{ or } 166\frac{2}{3}]\end{aligned}$$

The Budget rental will be a better deal than the U-Haul rental after 167 miles.

75. Cost $C = 20x + 100$; Revenue $R = 24x$. The business will show a profit only when $R > C$. Substitute the given expressions for R and C .

$$\begin{aligned}R &> C \\ 24x &> 20x + 100 \\ 4x &> 100 \\ x &> 25\end{aligned}$$

The company will show a profit upon selling 26 DVDs.

76. $C = 3x + 2300$, $R = 5.50x$. To make a profit, $R > C$.

$$\begin{aligned}5.50x &> 3x + 2300 \\ 2.5x &> 2300 \\ x &> 920\end{aligned}$$

The company will show a profit after making 921 deliveries.

77.
$$\text{BMI} = \frac{704 \times (\text{weight in pounds})}{(\text{height in inches})^2}$$

(a) Let the height equal 72.

$$19 \leq \text{BMI} \leq 25$$

$$19 \leq \frac{704w}{72^2} \leq 25$$

$$19(72^2) \leq 704w \leq 25(72^2)$$

$$\frac{19(72^2)}{704} \leq w \leq \frac{25(72^2)}{704}$$

$$(\approx 139.91) \leq w \leq (\approx 184.09)$$

According to the BMI formula, the healthy weight range (rounded to the nearest pound) for a person that is 72 inches tall is 140 to 184 pounds.

(b) Answers will vary.

78. (a) Let $A = 35$.

$$0.7(220 - A) \leq \text{THR} \leq 0.85(220 - A)$$

$$0.7(220 - 35) \leq \text{THR} \leq 0.85(220 - 35)$$

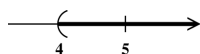
$$0.7(185) \leq \text{THR} \leq 0.85(185)$$

$$129.5 \leq \text{THR} \leq 157.25$$

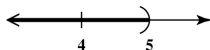
The range is about 130 to 157 beats per minute.

(b) Answers will vary.

79. (a) $x > 4$ is equivalent to $(4, \infty)$.

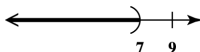


(b) $x < 5$ is equivalent to $(-\infty, 5)$.

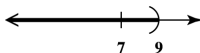


(c) All numbers greater than 4 and less than 5 (that is, $4 < x < 5$) belong to *both* sets.

80. (a) $m < 7$ is equivalent to $(-\infty, 7)$.

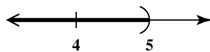


(b) $m < 9$ is equivalent to $(-\infty, 9)$.

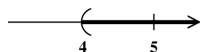


(c) All numbers less than 7 (that is, $m < 7$) belong to *both* sets.

81. (a) $t < 5$ is equivalent to $(-\infty, 5)$.

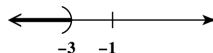


(b) $t > 4$ is equivalent to $(4, \infty)$.

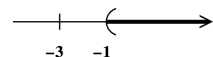


(c) All real numbers belong to *either one or both* of these sets.

82. (a) $s < -3$ is equivalent to $(-\infty, -3)$.



(b) $s > -1$ is equivalent to $(-1, \infty)$.



(c) All numbers greater than or equal to -3 , but less than or equal to -1 (that is, $-3 \leq s \leq -1$) belong to *neither* of these sets.

2.6 Set Operations and Compound Inequalities

2.6 Classroom Examples

1. Let $A = \{3, 4, 5, 6\}$ and $B = \{5, 6, 7\}$.

The set $A \cap B$, the intersection of A and B , contains those elements that belong to both A and B ; that is, the numbers 5 and 6. Therefore,

$$A \cap B = \{5, 6\}.$$

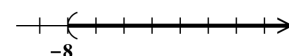
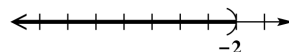
2. $x + 3 < 1$ and $x - 4 > -12$

Solve each inequality.

$$x + 3 < 1 \quad \text{and} \quad x - 4 > -12$$

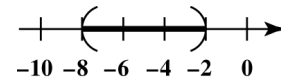
$$x + 3 - 3 < 1 - 3 \quad \text{and} \quad x - 4 + 4 > -12 + 4$$

$$x < -2 \quad \text{and} \quad x > -8$$



The values that satisfy both inequalities are the numbers between -8 and -2 , excluding -8 and -2 .

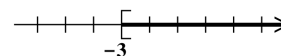
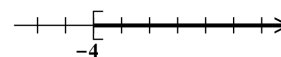
The solution set is $(-8, -2)$.



3. $2x \leq 4x + 8$ and $3x \geq -9$

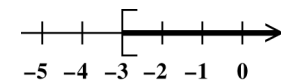
$$-2x \leq 8 \quad \text{and} \quad x \geq -3$$

$$x \geq -4$$



The overlap of the two graphs consists of the numbers that are greater than or equal to -4 and are also greater than or equal to -3 ; that is, the numbers greater than or equal to -3 .

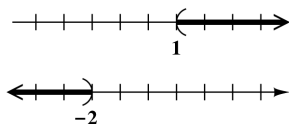
The solution set is $[-3, \infty)$.



4. $x + 2 > 3$ and $2x + 1 < -3$

$$2x < -4$$

$$x > 1 \text{ and } x < -2$$



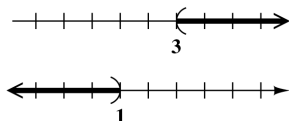
There is no number that is both greater than 1 *and* less than -2 , so the given compound inequality has no solution. The solution set is $\emptyset$.

5. Let $A = \{3, 4, 5, 6\}$ and $B = \{5, 6, 7\}$.

The set $A \cup B$, the union of A and B , consists of all elements in either A *or* B (or both). Start by listing the elements of set A : 3, 4, 5, 6. Then list any additional elements from set B . In this case, the elements 5 and 6 are already listed, so the only additional element is 7. Therefore,

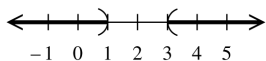
$$A \cup B = \{3, 4, 5, 6, 7\}.$$

6. $x - 1 > 2$ or $3x + 5 < 2x + 6$
 $x > 3$ or $x < 1$



The graph of the solution set consists of all numbers greater than 3 *or* less than 1.

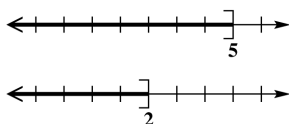
The solution set is $(-\infty, 1) \cup (3, \infty)$.



7. $3x - 2 \leq 13$ or $x + 5 \leq 7$

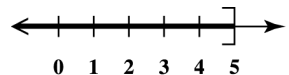
$$3x \leq 15$$

$$x \leq 5 \text{ or } x \leq 2$$



The solution set is all numbers that are either less than or equal to 5 *or* less than or equal to 2. All real numbers less than or equal to 5 are included.

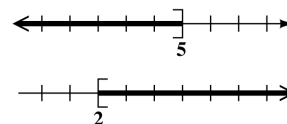
The solution set is $(-\infty, 5]$.



8. $3x - 2 \leq 13$ or $x + 5 \geq 7$

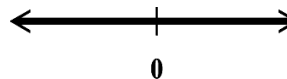
$$3x \leq 15$$

$$x \leq 5 \text{ or } x \geq 2$$



The solution set is all numbers that are either less than or equal to 5 *or* greater than or equal to 2. All real numbers are included.

The solution set is $(-\infty, \infty)$.



9. (a) All films had admissions greater than 130,000,000, but no films had a gross income of less than \$800,000,000. Thus, there are no elements (films) that satisfy each set, so the required set is the empty set, symbolized by $\emptyset$.

(b) Since all the films had admissions greater than 130,000,000 and we have an *or* statement, the second condition doesn't have an effect on the solution set. The required set is the set of all films; that is, $\{\text{Gone with the Wind, Star Wars, The Sound of Music, E.T., The Ten Commandments}\}$.

2.6 Section Exercises

- This statement is *true*. The solution set of $x + 1 = 5$ is $\{4\}$. The solution set of $x + 1 > 5$ is $(4, \infty)$. The solution set of $x + 1 < 5$ is $(-\infty, 4)$. Taken together we have the set of real numbers. (See Section 2.5, Exercises 43–47, for a discussion of this concept.)
- This statement is *false*. The intersection is $\{7\}$ since 7 is the element common to both sets.
- This statement is *false*. The union is $(-\infty, 8) \cup (8, \infty)$. The only real number that is *not* in the union is 8.
- This statement is *true* since 8 is the only element common to both sets.
- This statement is *false* since 0 is a rational number but not an irrational number. The sets of rational numbers and irrational numbers have no common elements so their intersection is $\emptyset$.
- This statement is *true*. The set of rational numbers together with the set of irrational numbers makes up the set of real numbers.

In Exercises 7–14, let

$A = \{1, 2, 3, 4, 5, 6\}$, $B = \{1, 3, 5\}$, $C = \{1, 6\}$,
and $D = \{4\}$.

7. The intersection of sets B and A contains only those elements in both sets B and A .

$$B \cap A = \{1, 3, 5\} \text{ or set } B$$

8. The intersection of sets A and B contains only those elements in both sets A and B .

$$A \cap B = \{1, 3, 5\} \text{ or set } B$$

Note that $A \cap B = B \cap A$.

9. The intersection of sets A and D is the set of all elements in both set A and D . Therefore,

$$A \cap D = \{4\} \text{ or set } D.$$

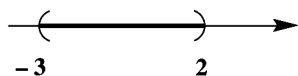
10. 1 is the only element in both sets B and C , so $B \cap C = \{1\}$.
11. The intersection of set B and the set of no elements (empty set), $B \cap \emptyset$, is the set of no elements or $\emptyset$.
12. No element is common to both sets, so $A \cap \emptyset = \emptyset$, the empty set.
13. The union of sets A and B is the set of all elements that are in either set A or set B or both sets A and B . Since all numbers in set B are also in set A , the set $A \cup B$ will be the same as set A .

$$A \cup B = \{1, 2, 3, 4, 5, 6\} \text{ or set } A$$

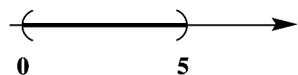
14. A union of sets contains all elements that belong to either set.

$$B \cup D = \{1, 3, 4, 5\}$$

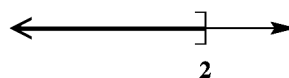
15. The first graph represents the set $(-\infty, 2)$. The second graph represents the set $(-3, \infty)$. The intersection includes the elements common to both sets, that is, $(-3, 2)$.



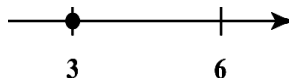
16. The first graph represents the set $(-\infty, 5)$. The second graph represents the set $(0, \infty)$. The intersection includes the elements common to both sets, that is, $(0, 5)$.



17. The first graph represents the set $(-\infty, 5]$. The second graph represents the set $(-\infty, 2]$. The intersection includes the elements common to both sets, that is, $(-\infty, 2]$.

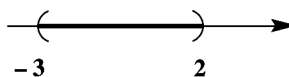


18. The first graph represents the set $[3, \infty)$. The second graph represents the set $(-\infty, 3]$. The intersection includes the elements common to both sets, that is, $\{3\}$.



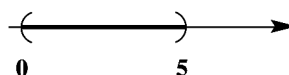
19. $x < 2$ and $x > -3$

The graph of the solution set will be all numbers that are both less than 2 and greater than -3 . The solution set is $(-3, 2)$.



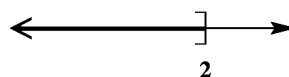
20. $x < 5$ and $x > 0$

And means intersection. The graph of the solution set will be all numbers that are both less than 5 and greater than 0. The overlap is the numbers between 0 and 5, not including 0 and 5. The solution set is $(0, 5)$.



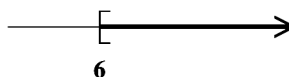
21. $x \leq 2$ and $x \leq 5$

The graph of the solution set will be all numbers that are both less than or equal to 2 and less than or equal to 5. The overlap is the numbers less than or equal to 2. The solution set is $(-\infty, 2]$.



22. $x \geq 3$ and $x \geq 6$

The graph of the solution set will be all numbers that are both greater than or equal to 3 and greater than or equal to 6. This will be all numbers greater than or equal to 6. The solution set is $[6, \infty)$.



23. $x \leq 3$ and $x \geq 6$

The graph of the solution set will be all numbers that are both less than or equal to 3 and greater than or equal to 6. There are no such numbers. The solution set is $\emptyset$.

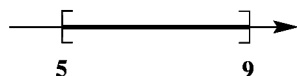
24. $x \leq -1$ and $x \geq 3$

The graph of the solution set will be all numbers that are both less than or equal to -1 and greater than or equal to 3 . There are no such numbers.

The solution set is $\emptyset$.

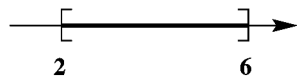
25. $x - 3 \leq 6$ and $x + 2 \geq 7$
 $x \leq 9$ and $x \geq 5$

The graph of the solution set is all numbers that are both less than or equal to 9 and greater than or equal to 5 . This is the intersection. The elements common to both sets are the numbers between 5 and 9 , including the endpoints. The solution set is $[5, 9]$.



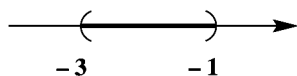
26. $x + 5 \leq 11$ and $x - 3 \geq -1$
 $x \leq 6$ and $x \geq 2$

The graph of the solution set is all numbers that are both less than or equal to 6 and greater than or equal to 2 . This is the intersection. The elements common to both sets are the numbers between 2 and 6 , including the endpoints. The solution set is $[2, 6]$.



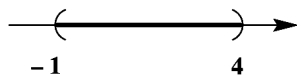
27. $-3x > 3$ and $x + 3 > 0$
 $x < -1$ and $x > -3$

The graph of the solution set is all numbers that are both less than -1 and greater than -3 . This is the intersection. The elements common to both sets are the numbers between -3 and -1 , not including the endpoints. The solution set is $(-3, -1)$.



28. $-3x < 3$ and $x + 2 < 6$
 $x > -1$ and $x < 4$

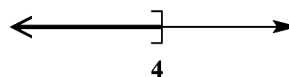
The graph of the solution set is all numbers that are both less than 4 and greater than -1 . This is the intersection. The elements common to both sets are the numbers between -1 and 4 , not including the endpoints. The solution set is $(-1, 4)$.



29. $3x - 4 \leq 8$ and $-4x + 1 \geq -15$
 $3x \leq 12$ and $-4x \geq -16$
 $x \leq 4$ and $x \leq 4$

Since both inequalities are identical, the graph of

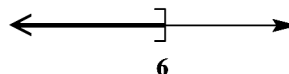
the solution set is the same as the graph of one of the inequalities. The solution set is $(-\infty, 4]$.



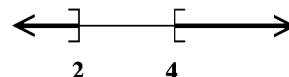
30. $7x + 6 \leq 48$ and $-4x \geq -24$
 $7x \leq 42$

$x \leq 6$ and $x \leq 6$

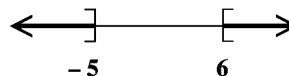
Since both inequalities are identical, the graph of the solution set is the same as the graph of one of the inequalities. The solution set is $(-\infty, 6]$.



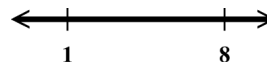
31. The first graph represents the set $(-\infty, 2]$. The second graph represents the set $[4, \infty)$. The union includes all elements in either set, or in both, that is, $(-\infty, 2] \cup [4, \infty)$.



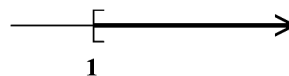
32. The first graph represents the set $(-\infty, -5]$. The second graph represents the set $[6, \infty)$. The union includes all elements in either set, or in both, that is, $(-\infty, -5] \cup [6, \infty)$.



33. The first graph represents the set $[1, \infty)$. The second graph represents the set $(-\infty, 8]$. The union includes all elements in either set, or in both, that is, $(-\infty, \infty)$.

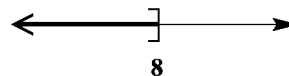


34. The first graph represents the set $[1, \infty)$. The second graph represents the set $[8, \infty)$. The union includes all elements in either set, or in both, that is, $[1, \infty)$.



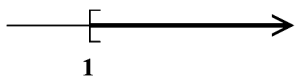
35. $x \leq 1$ or $x \leq 8$

The word "or" means to take the union of both sets. The graph of the solution set is all numbers that are either less than or equal to 1 or less than or equal to 8 , or both. This is all numbers less than or equal to 8 . The solution set is $(-\infty, 8]$.



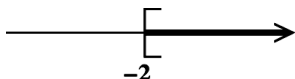
36. $x \geq 1$ or $x \geq 8$

The graph of the solution set will be all numbers that are either greater than or equal to 1 or greater than or equal to 8. The solution set is $[1, \infty)$.



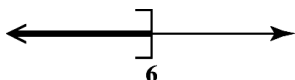
37. $x \geq -2$ or $x \geq 5$

The graph of the solution set will be all numbers that are either greater than or equal to -2 or greater than or equal to 5 . The solution set is $[-2, \infty)$.



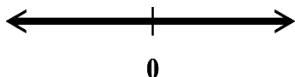
38. $x \leq -2$ or $x \leq 6$

The graph of the solution set will be all numbers that are either less than or equal to -2 or less than or equal to 6 . The solution set is $(-\infty, 6]$.



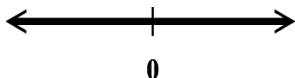
39. $x \geq -2$ or $x \leq 4$

The graph of the solution set will be all numbers that are either greater than or equal to -2 or less than or equal to 4 . This is the set of all real numbers. The solution set is $(-\infty, \infty)$.



40. $x \geq 5$ or $x \leq 7$

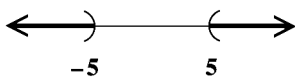
The graph of the solution set will be all numbers that are either greater than or equal to 5 or less than or equal to 7 . The solution set is $(-\infty, \infty)$.



41. $x + 2 > 7$ or $1 - x > 6$
 $-x > 5$

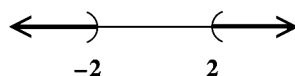
$x > 5$ or $x < -5$

The graph of the solution set is all numbers either greater than 5 or less than -5 . This is the union. The solution set is $(-\infty, -5) \cup (5, \infty)$.



42. $x + 1 > 3$ or $x + 4 < 2$
 $x > 2$ or $x < -2$

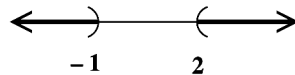
The graph of the solution set is all numbers either greater than 2 or less than -2 . This is the union. The solution set is $(-\infty, -2) \cup (2, \infty)$.



43. $x + 1 > 3$ or $-4x + 1 > 5$
 $-4x > 4$

$x > 2$ or $x < -1$

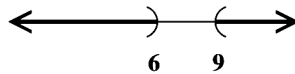
The graph of the solution set is all numbers either less than -1 or greater than 2 . This is the union. The solution set is $(-\infty, -1) \cup (2, \infty)$.



44. $3x < x + 12$ or $x + 1 > 10$
 $2x < 12$

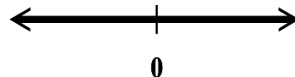
$x < 6$ or $x > 9$

The graph of the solution set is all numbers either less than 6 or greater than 9 . This is the union. The solution set is $(-\infty, 6) \cup (9, \infty)$.



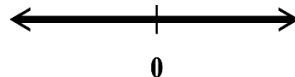
45. $4x + 1 \geq -7$ or $-2x + 3 \geq 5$
 $4x \geq -8$ or $-2x \geq 2$
 $x \geq -2$ or $x \leq -1$

The graph of the solution set is all numbers either greater than or equal to -2 or less than or equal to -1 . This is the set of all real numbers. The solution set is $(-\infty, \infty)$.



46. $3x + 2 \leq -7$ or $-2x + 1 \leq 9$
 $3x \leq -9$ or $-2x \leq 8$
 $x \leq -3$ or $x \geq -4$

The graph of the solution set is all numbers either less than or equal to -3 or greater than or equal to -4 . This is the set of all real numbers. The solution set is $(-\infty, \infty)$.



47. $(-\infty, -1] \cap [-4, \infty)$

The intersection is the set of numbers less than or equal to -1 and greater than or equal to -4 . The numbers common to both original sets are between, and including, -4 and -1 . The simplest interval form is $[-4, -1]$.

48. $[-1, \infty) \cap (-\infty, 9] = (-\infty, 9] \cap [-1, \infty)$

The intersection is the set of numbers less than or equal to 9 and greater than or equal to -1 . The numbers common to both original sets are between, and including, -1 and 9 . The simplest interval form is $[-1, 9]$.

49. $(-\infty, -6] \cap [-9, \infty)$

The intersection is the set of numbers less than or equal to -6 and greater than or equal to -9 . The numbers common to both original sets are between, and including, -9 and -6 . The simplest interval form is $[-9, -6]$.

50. $(5, 11] \cap [6, \infty)$

The intersection is the set of numbers between 5 and 11 , including 11 but not 5 , and greater than or equal to 6 . The numbers common to both original sets are between, and including, 6 and 11 . The simplest interval form is $[6, 11]$.

51. $(-\infty, 3) \cup (-\infty, -2)$

The union is the set of numbers that are either less than 3 or less than -2 , or both. This is all numbers less than 3 . The simplest interval form is $(-\infty, 3)$.

52. $[-9, 1] \cup (-\infty, -3)$

The union is the set of numbers between -9 and 1 , including both, or less than -3 . This is all numbers less than, and including 1 . The simplest interval form is $(-\infty, 1]$.

53. $[3, 6] \cup (4, 9)$

The union is the set of numbers between, and including, 3 and 6 , or between, but not including, 4 and 9 . This is the set of numbers greater than or equal to 3 and less than 9 . The simplest interval form is $[3, 9)$.

54. $[-1, 2] \cup (0, 5)$

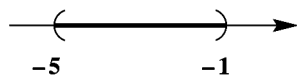
The union is the set of numbers between, and including, -1 and 2 , or between, but not including, 0 and 5 . This is the set of numbers greater than or equal to -1 and less than 5 . The simplest interval form is $[-1, 5)$.

55. $x < -1$ and $x > -5$

The word "and" means to take the intersection of both sets. $x < -1$ and $x > -5$ is true only when

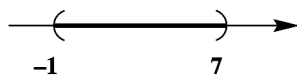
$$-5 < x < -1.$$

The graph of the solution set is all numbers greater than -5 and less than -1 . This is all numbers between -5 and -1 , not including -5 or -1 . The solution set is $(-5, -1)$.



56. $x > -1$ and $x < 7$

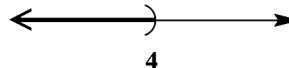
This is an intersection. The solution set is $(-1, 7)$.



57. $x < 4$ or $x < -2$

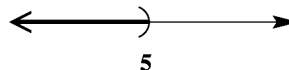
The word "or" means to take the union of both sets. The graph of the solution set is all numbers

that are either less than 4 or less than -2 , or both. This is all numbers less than 4 . The solution set is $(-\infty, 4)$.



58. $x < 5$ or $x < -3$

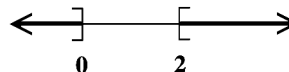
This is a union. The solution set is $(-\infty, 5)$.



59. $-3x \leq -6$ or $-3x \geq 0$

$$x \geq 2 \quad \text{or} \quad x \leq 0$$

The word "or" means to take the union of both sets. The graph of the solution set is all numbers that are either greater than or equal to 2 or less than or equal to 0 . The solution set is $(-\infty, 0] \cup [2, \infty)$.



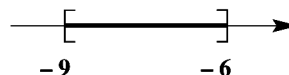
60. $2x - 6 \leq -18$ and $2x \geq -18$

$$2x \leq -12$$

$$x \leq -6 \quad \text{and} \quad x \geq -9$$

This is an intersection.

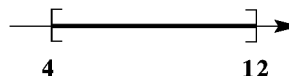
The solution set is $[-9, -6]$.



61. $x + 1 \geq 5$ and $x - 2 \leq 10$

$$x \geq 4 \quad \text{and} \quad x \leq 12$$

The word "and" means to take the intersection of both sets. The graph of the solution set is all numbers that are both greater than or equal to 4 and less than or equal to 12 . This is all numbers between, and including, 4 and 12 . The solution set is $[4, 12]$.

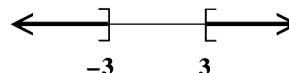


62. $-8x \leq -24$ or $-5x \geq 15$

$$x \geq 3 \quad \text{or} \quad x \leq -3$$

This is a union.

The solution set is $(-\infty, -3] \cup [3, \infty)$.



63. The set of expenses that are less than \$3000 for public schools and are greater than \$5000 for private schools is {Tuition and fees}.

64. The set of expenses that are less than \$2800 for public schools and are less than \$4000 for private schools is {Board rates}.

65. The set of expenses that are greater than \$2900 for public schools *or* are greater than \$5000 for private schools is {Tuition and fees, Dormitory charges}.

66. The set of expenses that are greater than \$4000 *or* are less than \$2700 is {Tuition and fees}.

67. Find "the yard can be fenced *and* the yard can be sodded."

A yard that can be fenced has $P \leq 150$. Maria and Joe qualify.

A yard that can be sodded has $A \leq 1400$. Again, Maria and Joe qualify.

Find the intersection. Maria's and Joe's yards are common to both sets, so Maria and Joe can have their yards both fenced and sodded.

68. Find "the yard can be fenced *and* the yard cannot be sodded."

A yard that can be fenced has $P \leq 150$. Maria and Joe qualify.

A yard that cannot be sodded has $A > 1400$. Luigi and Than qualify.

Find the intersection. There are no yards common to both sets, so none of them qualify.

69. Find "the yard cannot be fenced *and* the yard can be sodded."

A yard that cannot be fenced has $P > 150$. Luigi and Than qualify.

A yard that can be sodded has $A \leq 1400$. Maria and Joe qualify.

Find the intersection. There are no yards common to both sets, so none of the qualify.

70. Find "the yard cannot be fenced *and* the yard cannot be sodded."

A yard that cannot be fenced has $P > 150$. Luigi and Than qualify.

A yard that cannot be sodded has $A > 1400$. Again, Luigi and Than qualify.

Find the intersection. Luigi's and Than's yards are common to both sets, so Luigi and Than qualify.

71. Find "the yard can be fenced *or* the yard can be sodded." From Exercise 67, Maria's and Joe's yards qualify for both conditions, so the union is Maria and Joe.

72. Find "the yard cannot be fenced *or* the yard can be sodded." From Exercise 69, Luigi's and Than's yards cannot be fenced, and Maria's and Joe's yards can be sodded. The union includes all of them.

$$\begin{aligned} 73. \quad & 2y - 4 \leq 3y + 2 \\ & -y - 4 \leq 2 \\ & -y \leq 6 \\ & y \geq -6 \end{aligned}$$

The solution set is $[-6, \infty)$.

$$\begin{aligned} 74. \quad & 5t - 8 < 6t - 7 \\ & -t - 8 < -7 \\ & -t < 1 \\ & t > -1 \end{aligned}$$

The solution set is $(-1, \infty)$.

$$\begin{aligned} 75. \quad & -5 < 2r + 1 < 5 \\ & -6 < 2r < 4 \\ & -3 < r < 2 \end{aligned}$$

The solution set is $(-3, 2)$.

$$\begin{aligned} 76. \quad & -7 \leq 3w - 2 < 7 \\ & -5 \leq 3w < 9 \\ & -\frac{5}{3} \leq w < 3 \end{aligned}$$

The solution set is $[-\frac{5}{3}, 3)$.

$$\begin{aligned} 77. \quad & -|6| - |-11| + (-4) = -6 - (11) + (-4) \\ & = -17 + (-4) \\ & = -21 \end{aligned}$$

$$\begin{aligned} 78. \quad & (-5) - |-9| + |5 - 4| = -5 - (9) + 1 \\ & = -14 + 1 \\ & = -13 \end{aligned}$$

2.7 Absolute Value Equations and Inequalities

2.7 Classroom Examples

1. $|3x - 4| = 11$

$$\begin{aligned} 3x - 4 &= 11 & \text{or} & & 3x - 4 &= -11 \\ 3x &= 15 & \text{or} & & 3x &= -7 \\ x &= 5 & \text{or} & & x &= -\frac{7}{3} \end{aligned}$$

Check $x = 5$: $|11| = 11$ True

Check $x = -\frac{7}{3}$: $|-11| = 11$ True

The solution set is $\{-\frac{7}{3}, 5\}$.

2. $|3x - 4| \geq 11$

$$\begin{aligned} 3x - 4 &\geq 11 & \text{or} & & 3x - 4 &\leq -11 \\ 3x &\geq 15 & \text{or} & & 3x &\leq -7 \\ x &\geq 5 & \text{or} & & x &\leq -\frac{7}{3} \end{aligned}$$

Check $x = -3, 0$, and 6 in $|3x - 4| \geq 11$.

Check $x = -3$: $|-13| \geq 11$ True

Check $x = 0$: $|-4| \geq 11$ False

Check $x = 6$: $|14| \geq 11$ True

The solution set is $(-\infty, -\frac{7}{3}] \cup [5, \infty)$.

3. $|3x - 4| \leq 11$

$$-11 \leq 3x - 4 \leq 11$$

$$-7 \leq 3x \leq 15$$

$$-\frac{7}{3} \leq x \leq 5$$

Check $x = -3, 0$, and 6 in $|3x - 4| \leq 11$.

Check $x = -3$: $|-13| \leq 11$ *False*

Check $x = 0$: $|-4| \leq 11$ *True*

Check $x = 6$: $|14| \leq 11$ *False*

The solution set is $[-\frac{7}{3}, 5]$.

4A. $|3a + 2| + 4 = 15$

We first *isolate* the absolute value expression, that is, rewrite the equation so that the absolute value expression is alone on one side of the equals sign.

$$|3a + 2| = 11$$

$$3a + 2 = 11 \quad \text{or} \quad 3a + 2 = -11$$

$$3a = 9 \quad \text{or} \quad 3a = -13$$

$$a = 3 \quad \text{or} \quad a = -\frac{13}{3}$$

Check $a = 3$: $|11| + 4 = 15$ *True*

Check $a = -\frac{13}{3}$: $|-11| + 4 = 15$ *True*

The solution set is $\{-\frac{13}{3}, 3\}$.

4B. (a) $|x + 2| - 3 > 2$

$$|x + 2| > 5 \quad \text{Isolate.}$$

$$x + 2 > 5 \quad \text{or} \quad x + 2 < -5$$

$$x > 3 \quad \text{or} \quad x < -7$$

The solution set is $(-\infty, -7) \cup (3, \infty)$.

(b) $|3x + 2| + 4 \leq 15$

$$|3x + 2| \leq 11 \quad \text{Isolate.}$$

$$-11 \leq 3x + 2 \leq 11$$

$$-13 \leq 3x \leq 9$$

$$-\frac{13}{3} \leq x \leq 3$$

The solution set is $[-\frac{13}{3}, 3]$.

5. $|4r - 1| = |3r + 5|$

$$4r - 1 = 3r + 5 \quad \text{or} \quad 4r - 1 = -(3r + 5)$$

$$r = 6 \quad \text{or} \quad 4r - 1 = -3r - 5$$

$$7r = -4$$

$$r = -\frac{4}{7}$$

Check $r = 6$: $|23| = |23|$ *True*

Check $r = -\frac{4}{7}$: $|\frac{-23}{7}| = |\frac{23}{7}|$ *True*

The solution set is $\{-\frac{4}{7}, 6\}$.

6. (a) $|6x + 7| = -5$

Since the absolute value of an expression can never be negative, there are no solutions for this equation.

The solution set is $\emptyset$.

(b) $|\frac{1}{4}x - 3| = 0$

The expression $\frac{1}{4}x - 3$ will equal 0 *only* if

$$\frac{1}{4}x - 3 = 0.$$

$$x - 12 = 0 \quad \text{Multiply by 4.}$$

$$x = 12$$

The solution set is $\{12\}$.

7. (a) $|x| > -1$

The absolute value of a number is always greater than or equal to 0. Therefore, the inequality is true for all real numbers.

The solution set is $(-\infty, \infty)$.

(b) $|t - 10| - 2 \leq -3$

$$|t - 10| \leq -1$$

There is no number whose absolute value is less than a negative number, so this inequality has no solution.

The solution set is $\emptyset$.

(c) $|x + 2| \leq 0$

The value of $|x + 2|$ will never be less than 0. However, $|x + 2|$ will equal 0 when $x = -2$.The solution set is $\{-2\}$.

2.7 Section Exercises

- 1.
- $|x| = 5$
- has two solutions,
- $x = 5$
- or
- $x = -5$
- . The graph is Choice
- E**
- .

 $|x| < 5$ is written $-5 < x < 5$. Notice that -5 and 5 are not included. The graph is Choice **C**, which uses parentheses. $|x| > 5$ is written $x < -5$ or $x > 5$. The graph is Choice **D**, which uses parentheses. $|x| \leq 5$ is written $-5 \leq x \leq 5$. This time -5 and 5 are included. The graph is Choice **B**, which uses brackets. $|x| \geq 5$ is written $x \leq -5$ or $x \geq 5$. The graph is Choice **A**, which uses brackets.

- 2.
- $|x| = 9$
- has two solutions,
- $x = 9$
- and
- $x = -9$
- . The graph is Choice
- E**
- .

 $|x| > 9$ is written $x < -9$ or $x > 9$. Notice that -9 and 9 are not included. The graph is Choice **D**, which uses parentheses. $|x| \geq 9$ is written $x \leq -9$ or $x \geq 9$. This time -9 and 9 are included. The graph is Choice **A**, which uses brackets. $|x| < 9$ is written $-9 < x < 9$. The graph is Choice **C**, which uses parentheses. $|x| \leq 9$ is written $-9 \leq x \leq 9$. The graph is Choice **B**, which uses brackets.

3. (a) $|ax + b| = k, k = 0$

This means the distance from $ax + b$ to 0 is 0, so $ax + b = 0$, which has one solution.

(b) $|ax + b| = k, k > 0$

This means the distance from $ax + b$ to 0 is a positive number, so $ax + b = k$ or $ax + b = -k$. There are two solutions.

(c) $|ax + b| = k, k < 0$

This means the distance from $ax + b$ to 0 is a negative number, which is impossible because distance is always positive. There are no solutions.

4. When solving an absolute value equation or inequality of the form

$|ax + b| = k, |ax + b| < k, \text{ or } |ax + b| > k,$

where k is a positive number, use *or* for the equality statement and the $>$ statement. Use *and* for the $<$ statement.

5. $|x| = 12$

$x = 12 \text{ or } x = -12$

The solution set is $\{-12, 12\}$.

6. $|k| = 14$

$k = 14 \text{ or } k = -14$

The solution set is $\{-14, 14\}$.

7. $|4x| = 20$

$4x = 20 \text{ or } 4x = -20$

$x = 5 \text{ or } x = -5$

The solution set is $\{-5, 5\}$.

8. $|5x| = 30$

$5x = 30 \text{ or } 5x = -30$

$x = 6 \text{ or } x = -6$

The solution set is $\{-6, 6\}$.

9. $|y - 3| = 9$

$y - 3 = 9 \text{ or } y - 3 = -9$

$y = 12 \text{ or } y = -6$

The solution set is $\{-6, 12\}$.

10. $|p - 5| = 13$

$p - 5 = 13 \text{ or } p - 5 = -13$

$p = 18 \text{ or } p = -8$

The solution set is $\{-8, 18\}$.

11. $|2x - 1| = 11$

$2x - 1 = 11 \text{ or } 2x - 1 = -11$

$2x = 12 \quad 2x = -10$

$x = 6 \text{ or } x = -5$

The solution set is $\{-5, 6\}$.

12. $|2y + 3| = 19$

$2y + 3 = 19 \text{ or } 2y + 3 = -19$

$2y = 16 \quad 2y = -22$

$y = 8 \text{ or } y = -11$

The solution set is $\{-11, 8\}$.

13. $|4r - 5| = 17$

$4r - 5 = 17 \text{ or } 4r - 5 = -17$

$4r = 22 \quad 4r = -12$

$r = \frac{22}{4} = \frac{11}{2} \text{ or } r = -3$

The solution set is $\{-3, \frac{11}{2}\}$.

14. $|5t - 1| = 21$

$5t - 1 = 21 \text{ or } 5t - 1 = -21$

$5t = 22 \quad 5t = -20$

$t = \frac{22}{5} \text{ or } t = -4$

The solution set is $\{-4, \frac{22}{5}\}$.

15. $|2y + 5| = 14$

$2y + 5 = 14 \text{ or } 2y + 5 = -14$

$2y = 9 \quad 2y = -19$

$y = \frac{9}{2} \text{ or } y = -\frac{19}{2}$

The solution set is $\{-\frac{19}{2}, \frac{9}{2}\}$.

16. $|2x - 9| = 18$

$2x - 9 = 18 \text{ or } 2x - 9 = -18$

$2x = 27 \quad 2x = -9$

$x = \frac{27}{2} \text{ or } x = -\frac{9}{2}$

The solution set is $\{-\frac{9}{2}, \frac{27}{2}\}$.

17. $|\frac{1}{2}x + 3| = 2$

$\frac{1}{2}x + 3 = 2 \text{ or } \frac{1}{2}x + 3 = -2$

$\frac{1}{2}x = -1 \quad \frac{1}{2}x = -5$

$x = -2 \text{ or } x = -10$

The solution set is $\{-10, -2\}$.

18. $|\frac{2}{3}q - 1| = 5$

$\frac{2}{3}q - 1 = 5 \text{ or } \frac{2}{3}q - 1 = -5$

$\frac{2}{3}q = 6 \quad \frac{2}{3}q = -4$

$q = \frac{3}{2}(6) \quad q = \frac{3}{2}(-4)$

$q = 9 \text{ or } q = -6$

The solution set is $\{-6, 9\}$.

19. $|1 + \frac{3}{4}k| = 7$

$1 + \frac{3}{4}k = 7 \text{ or } 1 + \frac{3}{4}k = -7$

Multiply each side by 4.

$4 + 3k = 28 \text{ or } 4 + 3k = -28$

$3k = 24 \quad 3k = -32$

$k = 8 \text{ or } k = -\frac{32}{3}$

The solution set is $\{-\frac{32}{3}, 8\}$.

20. $|2 - \frac{5}{2}m| = 14$

$2 - \frac{5}{2}m = 14 \text{ or } 2 - \frac{5}{2}m = -14$

$-\frac{5}{2}m = 12 \quad -\frac{5}{2}m = -16$

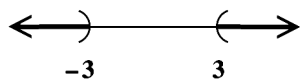
$m = (-\frac{2}{5})(12) \quad m = (-\frac{2}{5})(-16)$

$m = -\frac{24}{5} \text{ or } m = \frac{32}{5}$

The solution set is $\{-\frac{24}{5}, \frac{32}{5}\}$.

21. $|x| > 3$

$x > 3$ or $x < -3$

The solution set is $(-\infty, -3) \cup (3, \infty)$.

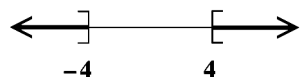
22. $|y| > 5$

$y < -5$ or $y > 5$

The solution set is $(-\infty, -5) \cup (5, \infty)$.

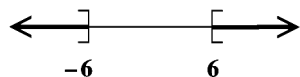
23. $|k| \geq 4$

$k \geq 4$ or $k \leq -4$

The solution set is $(-\infty, -4] \cup [4, \infty)$.

24. $|r| \geq 6$

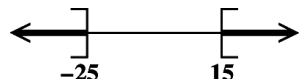
$r \leq -6$ or $r \geq 6$

The solution set is $(-\infty, -6] \cup [6, \infty)$.

25. $|r + 5| \geq 20$

$r + 5 \leq -20$ or $r + 5 \geq 20$

$r \leq -25$ or $r \geq 15$

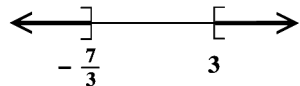
The solution set is $(-\infty, -25] \cup [15, \infty)$.

26. $|3x - 1| \geq 8$

$3x - 1 \geq 8$ or $3x - 1 \leq -8$

$3x \geq 9$ or $3x \leq -7$

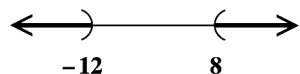
$x \geq 3$ or $x \leq -\frac{7}{3}$

The solution set is $(-\infty, -\frac{7}{3}] \cup [3, \infty)$.

27. $|t + 2| > 10$

$t + 2 > 10$ or $t + 2 < -10$

$t > 8$ or $t < -12$

The solution set is $(-\infty, -12) \cup (8, \infty)$.

28. $|4x + 1| \geq 21$

$4x + 1 \geq 21$ or $4x + 1 \leq -21$

$4x \geq 20$ or $4x \leq -22$

$x \geq 5$ or $x \leq -\frac{11}{2}$

The solution set is $(-\infty, -\frac{11}{2}] \cup [5, \infty)$.

29. $|3 - x| > 5$

$3 - x > 5$ or $3 - x < -5$

$-x > 2$ or $-x < -8$

Multiply by -1 ,

and reverse the inequality symbols.

$x < -2$ or $x > 8$

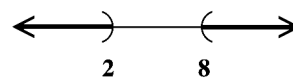
The solution set is $(-\infty, -2) \cup (8, \infty)$.

30. $|5 - x| > 3$

$5 - x > 3$ or $5 - x < -3$

$-x > -2$ or $-x < -8$

$x < 2$ or $x > 8$

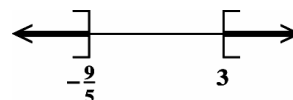
The solution set is $(-\infty, 2) \cup (8, \infty)$.

31. $|-5x + 3| \geq 12$

$-5x + 3 \geq 12$ or $-5x + 3 \leq -12$

$-5x \geq 9$ or $-5x \leq -15$

$x \leq -\frac{9}{5}$ or $x \geq 3$

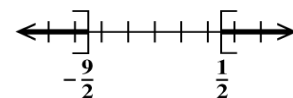
The solution set is $(-\infty, -\frac{9}{5}] \cup [3, \infty)$.

32. $|-2x - 4| \geq 5$

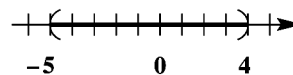
$-2x - 4 \geq 5$ or $-2x - 4 \leq -5$

$-2x \geq 9$ or $-2x \leq -1$

$x \leq -\frac{9}{2}$ or $x \geq \frac{1}{2}$

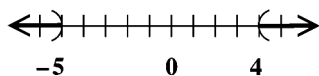
The solution set is $(-\infty, -\frac{9}{2}] \cup [\frac{1}{2}, \infty)$.

33. (a) $|2x + 1| < 9$

The graph of the solution set will be all numbers between -5 and 4 , since the absolute value is less than 9 .

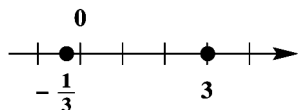
(b) $|2x + 1| > 9$

The graph of the solution set will be all numbers less than -5 or greater than 4 , since the absolute value is greater than 9 .



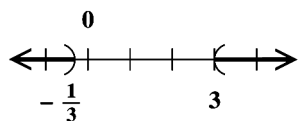
34. (a) $|3x - 4| = 5$

The solutions are the numbers at the endpoints in the given graph, $-\frac{1}{3}$ and 3 .



(b) $|3x - 4| > 5$

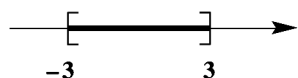
The solution set is composed of the numbers not in the given graph, not including the endpoints.



35. $|x| \leq 3$

$-3 \leq x \leq 3$

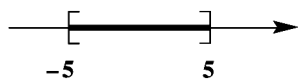
The solution set is $[-3, 3]$.



36. $|y| \leq 5$

$-5 \leq y \leq 5$

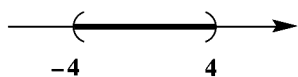
The solution set is $[-5, 5]$.



37. $|k| < 4$

$-4 < k < 4$

The solution set is $(-4, 4)$.



38. $|r| < 6$

$-6 < r < 6$

The solution set is $(-6, 6)$.

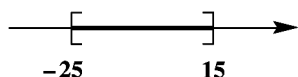


39. $|r + 5| \leq 20$

$-20 \leq r + 5 \leq 20$

$-25 \leq r \leq 15$ Subtract 5.

The solution set is $[-25, 15]$.



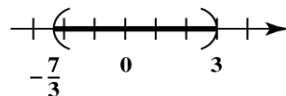
40. $|3x - 1| < 8$

$-8 < 3x - 1 < 8$

$-7 < 3x < 9$ Add 1.

$-\frac{7}{3} < x < 3$ Divide by 3.

The solution set is $(-\frac{7}{3}, 3)$.



41. $|t + 2| \leq 10$

$-10 \leq t + 2 \leq 10$

$-12 \leq t \leq 8$

The solution set is $[-12, 8]$.



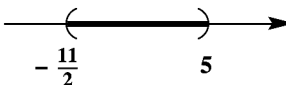
42. $|4x + 1| < 21$

$-21 < 4x + 1 < 21$

$-22 < 4x < 20$

$-\frac{11}{2} < x < 5$

The solution set is $(-\frac{11}{2}, 5)$.



43. $|3 - x| \leq 5$

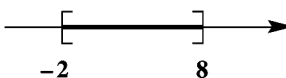
$-5 \leq 3 - x \leq 5$

$-8 \leq -x \leq 2$

Multiply by -1 , and reverse the inequality symbols.

$8 \geq x \geq -2$ or $-2 \leq x \leq 8$

The solution set is $[-2, 8]$.



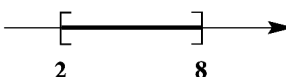
44. $|5 - x| \leq 3$

$-3 \leq 5 - x \leq 3$

$-8 \leq -x \leq -2$

$8 \geq x \geq 2$ or $2 \leq x \leq 8$

The solution set is $[2, 8]$.



45. $|-5x + 3| \leq 12$

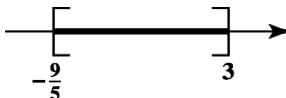
$-12 \leq -5x + 3 \leq 12$

$-15 \leq -5x \leq 9$

Divide by -5 , and reverse the inequality symbols.

$3 \geq x \geq -\frac{9}{5}$ or $-\frac{9}{5} \geq x \geq 3$

The solution set is $[-\frac{9}{5}, 3]$.

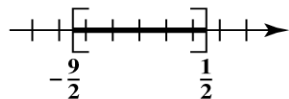


46. $|-2x - 4| \leq 5$

$$-5 \leq -2x - 4 \leq 5$$

$$-1 \leq -2x \leq 9$$

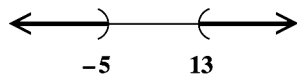
$$\frac{1}{2} \geq x \geq -\frac{9}{2} \quad \text{or} \quad -\frac{9}{2} \leq x \leq \frac{1}{2}$$

The solution set is $[-\frac{9}{2}, \frac{1}{2}]$.

47. $|-4 + k| > 9$

$$-4 + k > 9 \quad \text{or} \quad -4 + k < -9$$

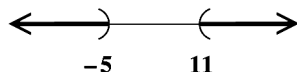
$$k > 13 \quad \text{or} \quad k < -5$$

The solution set is $(-\infty, -5) \cup (13, \infty)$.

48. $|-3 + t| > 8$

$$-3 + t > 8 \quad \text{or} \quad -3 + t < -8$$

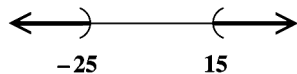
$$t > 11 \quad \text{or} \quad t < -5$$

The solution set is $(-\infty, -5) \cup (11, \infty)$.

49. $|r + 5| > 20$

$$r + 5 > 20 \quad \text{or} \quad r + 5 < -20$$

$$r > 15 \quad \text{or} \quad r < -25$$

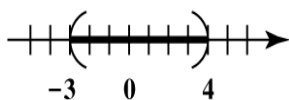
The solution set is $(-\infty, -25) \cup (15, \infty)$.

50. $|2x - 1| < 7$

$$-7 < 2x - 1 < 7$$

$$-6 < 2x < 8$$

$$-3 < x < 4$$

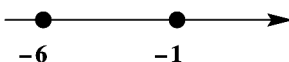
The solution set is $(-3, 4)$.

51. $|7 + 2z| = 5$

$$7 + 2z = 5 \quad \text{or} \quad 7 + 2z = -5$$

$$2z = -2 \quad \text{or} \quad 2z = -12$$

$$z = -1 \quad \text{or} \quad z = -6$$

The solution set is $\{-6, -1\}$.

52. $|9 - 3p| = 3$

$$9 - 3p = 3 \quad \text{or} \quad 9 - 3p = -3$$

$$-3p = -6 \quad \text{or} \quad -3p = -12$$

$$p = 2 \quad \text{or} \quad p = 4$$

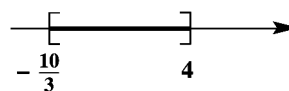
The solution set is $\{2, 4\}$.

53. $|3r - 1| \leq 11$

$$-11 \leq 3r - 1 \leq 11$$

$$-10 \leq 3r \leq 12$$

$$-\frac{10}{3} \leq r \leq 4$$

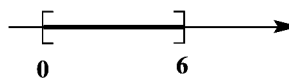
The solution set is $[-\frac{10}{3}, 4]$.

54. $|2s - 6| \leq 6$

$$-6 \leq 2s - 6 \leq 6$$

$$0 \leq 2s \leq 12$$

$$0 \leq s \leq 6$$

The solution set is $[0, 6]$.

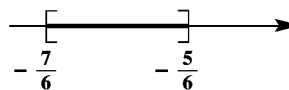
55. $|-6x - 6| \leq 1$

$$-1 \leq -6x - 6 \leq 1$$

$$5 \leq -6x \leq 7$$

Divide by -6 , and reverse the inequality symbols.

$$-\frac{5}{6} \geq x \geq -\frac{7}{6} \quad \text{or} \quad -\frac{7}{6} \leq x \leq -\frac{5}{6}$$

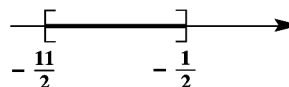
The solution set is $[-\frac{7}{6}, -\frac{5}{6}]$.

56. $|-2x - 6| \leq 5$

$$-5 \leq -2x - 6 \leq 5$$

$$1 \leq -2x \leq 11$$

$$-\frac{1}{2} \geq x \geq -\frac{11}{2} \quad \text{or} \quad -\frac{11}{2} \leq x \leq -\frac{1}{2}$$

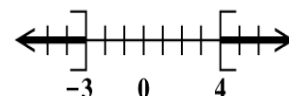
The solution set is $[-\frac{11}{2}, -\frac{1}{2}]$.

57. $|2x - 1| \geq 7$

$$2x - 1 \geq 7 \quad \text{or} \quad 2x - 1 \leq -7$$

$$2x \geq 8 \quad \text{or} \quad 2x \leq -6$$

$$x \geq 4 \quad \text{or} \quad x \leq -3$$

The solution set is $(-\infty, -3] \cup [4, \infty)$.

58. $|-4 + k| \leq 9$

$$\begin{aligned} -9 &\leq -4 + k \leq 9 \\ -5 &\leq k \leq 13 \end{aligned}$$

The solution set is $[-5, 13]$.



59. $|x + 2| = 3$

$$\begin{aligned} x + 2 &= 3 & \text{or} & & x + 2 &= -3 \\ x &= 1 & \text{or} & & x &= -5 \end{aligned}$$

The solution set is $\{-5, 1\}$.

60. $|x + 3| = 10$

$$\begin{aligned} x + 3 &= 10 & \text{or} & & x + 3 &= -10 \\ x &= 7 & \text{or} & & x &= -13 \end{aligned}$$

The solution set is $\{-13, 7\}$.

61. $|x - 6| = 3$

$$\begin{aligned} x - 6 &= 3 & \text{or} & & x - 6 &= -3 \\ x &= 9 & \text{or} & & x &= 3 \end{aligned}$$

The solution set is $\{3, 9\}$.

62. $|x - 4| = 1$

$$\begin{aligned} x - 4 &= 1 & \text{or} & & x - 4 &= -1 \\ x &= 5 & \text{or} & & x &= 3 \end{aligned}$$

The solution set is $\{3, 5\}$.

63. $|x| - 1 = 4$

$$|x| = 5$$

$$x = 5 \quad \text{or} \quad x = -5$$

The solution set is $\{-5, 5\}$.

64. $|x| + 3 = 10$

$$|x| = 7$$

$$x = 7 \quad \text{or} \quad x = -7$$

The solution set is $\{-7, 7\}$.

65. $|x + 4| + 1 = 2$

$$|x + 4| = 1$$

$$\begin{aligned} x + 4 &= 1 & \text{or} & & x + 4 &= -1 \\ x &= -3 & \text{or} & & x &= -5 \end{aligned}$$

The solution set is $\{-5, -3\}$.

66. $|x + 5| - 2 = 12$

$$|x + 5| = 14$$

$$\begin{aligned} x + 5 &= 14 & \text{or} & & x + 5 &= -14 \\ x &= 9 & \text{or} & & x &= -19 \end{aligned}$$

The solution set is $\{-19, 9\}$.

67. $|2x + 1| + 3 > 8$

$$|2x + 1| > 5$$

$$2x + 1 > 5 \quad \text{or} \quad 2x + 1 < -5$$

$$2x > 4 \quad \quad \quad 2x < -6$$

$$x > 2 \quad \text{or} \quad x < -3$$

The solution set is $(-\infty, -3) \cup (2, \infty)$.

68. $|6x - 1| - 2 > 6$

$$|6x - 1| > 8$$

$$6x - 1 > 8 \quad \text{or} \quad 6x - 1 < -8$$

$$6x > 9 \quad \quad \quad 6x < -7$$

$$x > \frac{3}{2} \quad \text{or} \quad x < -\frac{7}{6}$$

The solution set is $(-\infty, -\frac{7}{6}) \cup (\frac{3}{2}, \infty)$.

69. $|x + 5| - 6 \leq -1$

$$|x + 5| \leq 5$$

$$-5 \leq x + 5 \leq 5$$

$$-10 \leq x \leq 0$$

The solution set is $[-10, 0]$.

70. $|r - 2| - 3 \leq 4$

$$|r - 2| \leq 7$$

$$-7 \leq r - 2 \leq 7$$

$$-5 \leq r \leq 9$$

The solution set is $[-5, 9]$.

71. $|3x + 1| = |2x + 4|$

$$3x + 1 = 2x + 4 \quad \text{or} \quad 3x + 1 = -(2x + 4)$$

$$3x + 1 = -2x - 4$$

$$5x = -5$$

$$x = 3 \quad \quad \text{or} \quad \quad x = -1$$

The solution set is $\{-1, 3\}$.

72. $|7x + 12| = |x - 8|$

$$7x + 12 = x - 8 \quad \text{or} \quad 7x + 12 = -(x - 8)$$

$$6x = -20 \quad \quad \quad 7x + 12 = -x + 8$$

$$8x = -4$$

$$x = -\frac{10}{3} \quad \text{or} \quad x = -\frac{1}{2}$$

The solution set is $\{-\frac{10}{3}, -\frac{1}{2}\}$.

73. $|m - \frac{1}{2}| = |\frac{1}{2}m - 2|$

$$m - \frac{1}{2} = \frac{1}{2}m - 2 \quad \text{or} \quad m - \frac{1}{2} = -(\frac{1}{2}m - 2)$$

Multiply by 2. $m - \frac{1}{2} = -\frac{1}{2}m + 2$

$$2m - 1 = m - 4 \quad \quad \quad 2m - 1 = -m + 4$$

$$3m = 5$$

$$m = -3 \quad \quad \text{or} \quad \quad m = \frac{5}{3}$$

The solution set is $\{-3, \frac{5}{3}\}$.

74. $|\frac{2}{3}r - 2| = |\frac{1}{3}r + 3|$

$$\frac{2}{3}r - 2 = \frac{1}{3}r + 3 \quad \text{or} \quad \frac{2}{3}r - 2 = -(\frac{1}{3}r + 3)$$

$$\frac{1}{3}r = 5 \quad \quad \quad \frac{2}{3}r - 2 = -\frac{1}{3}r - 3$$

$$r = 15 \quad \quad \text{or} \quad \quad r = -1$$

The solution set is $\{-1, 15\}$.

75. $|6x| = |9x + 1|$

$$6x = 9x + 1 \quad \text{or} \quad 6x = -(9x + 1)$$

$$-3x = 1 \quad \quad \quad 6x = -9x - 1$$

$$15x = -1$$

$$x = -\frac{1}{15} \quad \text{or} \quad x = -\frac{1}{15}$$

The solution set is $\{-\frac{1}{15}, -\frac{1}{15}\}$.

76. $|13x| = |2x + 1|$

$$13x = 2x + 1 \quad \text{or} \quad 13x = -(2x + 1)$$

$$11x = 1 \quad \quad \quad 13x = -2x - 1$$

$$15x = -1$$

$$x = \frac{1}{11} \quad \text{or} \quad x = -\frac{1}{15}$$

The solution set is $\{-\frac{1}{15}, \frac{1}{11}\}$.

77. $|2p - 6| = |2p + 11|$

$$2p - 6 = 2p + 11 \quad \text{or} \quad 2p - 6 = -(2p + 11)$$

$$-6 = 11 \quad \text{False} \quad \quad \quad 2p - 6 = -2p - 11$$

$$4p = -5$$

$$\text{No solution} \quad \text{or} \quad p = -\frac{5}{4}$$

The solution set is $\{-\frac{5}{4}\}$.

78. $|3x - 1| = |3x + 9|$

$$3x - 1 = 3x + 9 \quad \text{or} \quad 3x - 1 = -(3x + 9)$$

$$-1 = 9 \quad \text{False} \quad \quad \quad 3x - 1 = -3x - 9$$

$$6x = -8$$

$$\text{No solution} \quad \text{or} \quad x = -\frac{4}{3}$$

The solution set is $\{-\frac{4}{3}\}$.

79. $|x| \geq -10$

The absolute value of a number is always greater than or equal to 0. Therefore, the inequality is true for all real numbers.

The solution set is $(-\infty, \infty)$.

80. $|x| \geq -15$

The absolute value of a number is always greater than or equal to 0. Therefore, the inequality is true for all real numbers.

The solution set is $(-\infty, \infty)$.

81. $|12t - 3| = -8$

Since the absolute value of an expression can never be negative, there are no solutions for this equation.

The solution set is $\emptyset$.

82. $|13w + 1| = -3$

Since the absolute value of an expression can never be negative, there are no solutions for this equation.

The solution set is $\emptyset$.

83. $|4x + 1| = 0$

The expression $4x + 1$ will equal 0 *only* for the solution of the equation

$$4x + 1 = 0.$$

$$4x = -1$$

$$x = -\frac{1}{4} \quad \text{or} \quad -\frac{1}{4}$$

The solution set is $\{-\frac{1}{4}\}$.

84. $|6r - 2| = 0$

$$6r - 2 = 0$$

$$6r = 2$$

$$r = \frac{1}{3}$$

The solution set is $\{\frac{1}{3}\}$.

85. $|2q - 1| = -6$

Since the absolute value of an expression can never be negative, there are no solutions for this equation.

The solution set is $\emptyset$.

86. $|8n + 4| = -4$

Since the absolute value of an expression can never be negative, there are no solutions for this equation.

The solution set is $\emptyset$.

87. $|x + 5| > -9$

Since the absolute value of an expression is always nonnegative (positive or zero), the inequality is true for any real number x .

The solution set is $(-\infty, \infty)$.

88. $|x + 9| > -3$

Since the absolute value of an expression is always nonnegative (positive or zero), the inequality is true for any real number x .

The solution set is $(-\infty, \infty)$.

89. $|7x + 3| \leq 0$

The absolute value of an expression is always nonnegative (positive or zero), so this inequality is true only when

$$7x + 3 = 0$$

$$7x = -3$$

$$x = -\frac{3}{7}$$

The solution set is $\{-\frac{3}{7}\}$.

90. $|4x - 1| \leq 0$

The absolute value of an expression is always nonnegative (positive or zero), so this inequality is true only when

$$\begin{aligned} 4x - 1 &= 0 \\ 4x &= 1 \\ x &= \frac{1}{4}. \end{aligned}$$

The solution set is $\{\frac{1}{4}\}$.

91. $|5x - 2| = 0$

The expression $5x - 2$ will equal 0 *only* for the solution of the equation

$$\begin{aligned} 5x - 2 &= 0. \\ 5x &= 2 \\ x &= \frac{2}{5} \end{aligned}$$

The solution set is $\{\frac{2}{5}\}$.

92. $|4 + 7x| = 0$

$$\begin{aligned} 4 + 7x &= 0 \\ 7x &= -4 \\ x &= -\frac{4}{7} \end{aligned}$$

The solution set is $\{-\frac{4}{7}\}$.

93. $|x - 2| + 3 \geq 2$

$$|x - 2| \geq -1$$

Since the absolute value of an expression is always nonnegative (positive or zero), the inequality is true for any real number x .

The solution set is $(-\infty, \infty)$.

94. $|k - 4| + 5 \geq 4$

$$|k - 4| \geq -1$$

Since the absolute value of an expression is always nonnegative (positive or zero), the inequality is true for any real number x .

The solution set is $(-\infty, \infty)$.

95. $|10z + 7| + 3 < 1$

$$|10z + 7| < -2$$

There is no number whose absolute value is less than -2 , so this inequality has no solution.

The solution set is $\emptyset$.

96. $|4x + 1| - 2 < -5$

$$|4x + 1| < -3$$

There is no number whose absolute value is less than -3 , so this inequality has no solution.

The solution set is $\emptyset$.

97. Let x represent the calcium intake for a specific female. For x to be within 100 mg of 1000 mg, we must have

$$\begin{aligned} |x - 1000| &\leq 100. \\ -100 &\leq x - 1000 \leq 100 \\ 900 &\leq x \leq 1100 \end{aligned}$$

98. Let x represent the clotting time for an individual. For x to be within 3.6 seconds of 7.45 seconds, we must have

$$\begin{aligned} |x - 7.45| &\leq 3.6 \\ -3.6 &\leq x - 7.45 \leq 3.6 \\ 3.85 &\leq x \leq 11.05 \end{aligned}$$

99. Add the given heights with a calculator to get 4756. There are 10 numbers, so divide the sum by 10.

$$\frac{4756}{10} = 475.6$$

The average height is 475.6 ft.

100. $|x - k| < 50$

Substitute 475.6 for k and solve the inequality.

$$\begin{aligned} |x - 475.6| &< 50 \\ -50 &< x - 475.6 < 50 \\ 425.6 &< x < 525.6 \end{aligned}$$

The buildings with heights between 425.6 ft and 525.6 ft are the 1201 Walnut, City Hall, Fidelity Bank and Trust Building, Kansas City Power and Light, and the Hyatt Regency Crown Center.

101. $|x - k| < 75$

Substitute 475.6 for k and solve the inequality.

$$\begin{aligned} |x - 475.6| &< 75 \\ -75 &< x - 475.6 < 75 \\ 400.6 &< x < 550.6 \end{aligned}$$

The buildings with heights between 400.6 ft and 550.6 ft are City Center Square, Commerce Tower, Federal Office Building, 1201 Walnut, City Hall, Fidelity Bank and Trust Building, Kansas City Power and Light, and the Hyatt Regency Crown Center.

102. (a) This would be the opposite of the inequality in Exercise 101, that is,

$$|x - 475.6| \geq 75.$$

(b) $|x - 475.6| \geq 75$

$$\begin{aligned} x - 475.6 &\geq 75 & \text{or} & & x - 475.6 &\leq -75 \\ x &\geq 550.6 & \text{or} & & x &\leq 400.6 \end{aligned}$$

(c) The buildings that are not within 75 ft of the average have height less than or equal to 400.6 or greater than or equal to 550.6. They are Town Pavillion and One Kansas City Place.

(d) The answer makes sense because it includes all the buildings *not* listed earlier which had heights within 75 ft of the average.

103. (a) $3x + 2y = 24$

$$3(0) + 2y = 24 \quad \text{Let } x = 0.$$

$$0 + 2y = 24$$

$$2y = 24$$

$$y = 12$$

(b) $-2x + 5y = 20$

$$-2(0) + 5y = 20 \quad \text{Let } x = 0.$$

$$0 + 5y = 20$$

$$5y = 20$$

$$y = 4$$

104. (a) $3x + 2y = 24$

$$3(-2) + 2y = 24 \quad \text{Let } x = -2.$$

$$-6 + 2y = 24$$

$$2y = 30$$

$$y = 15$$

(b) $-2x + 5y = 20$

$$-2(-2) + 5y = 20 \quad \text{Let } x = -2.$$

$$4 + 5y = 20$$

$$5y = 16$$

$$y = \frac{16}{5}$$

105. (a) $3x + 2y = 24$

$$3(8) + 2y = 24 \quad \text{Let } x = 8.$$

$$24 + 2y = 24$$

$$2y = 0$$

$$y = 0$$

(b) $-2x + 5y = 20$

$$-2(8) + 5y = 20 \quad \text{Let } x = 8.$$

$$-16 + 5y = 20$$

$$5y = 36$$

$$y = \frac{36}{5}$$

106. (a) $3x + 2y = 24$

$$3(1.5) + 2y = 24 \quad \text{Let } x = 1.5.$$

$$4.5 + 2y = 24$$

$$2y = 19.5$$

$$y = 9.75$$

(b) $-2x + 5y = 20$

$$-2(1.5) + 5y = 20 \quad \text{Let } x = 1.5.$$

$$-3 + 5y = 20$$

$$5y = 23$$

$$y = \frac{23}{5}, \text{ or } 4.6$$

Summary Exercises on Solving Linear and Absolute Value Equations and Inequalities

1. $4z + 1 = 49$

$$4z = 48$$

$$z = 12$$

The solution set is $\{12\}$.

2. $|m - 1| = 6$

$$m - 1 = 6 \quad \text{or} \quad m - 1 = -6$$

$$m = 7 \quad \text{or} \quad m = -5$$

The solution set is $\{-5, 7\}$.

3. $6q - 9 = 12 + 3q$

$$3q = 21$$

$$q = 7$$

The solution set is $\{7\}$.

4. $3p + 7 = 9 + 8p$

$$-5p = 2$$

$$p = -\frac{2}{5}$$

The solution set is $\{-\frac{2}{5}\}$.

5. $|a + 3| = -4$

Since the absolute value of an expression is always nonnegative, there is no number that makes this statement true. Therefore, the solution set is $\emptyset$.

6. $2m + 1 \leq m$

$$m \leq -1$$

The solution set is $(-\infty, -1]$.

7. $8r + 2 \geq 5r$

$$3r \geq -2$$

$$r \geq -\frac{2}{3}$$

The solution set is $[-\frac{2}{3}, \infty)$.

8. $4(a - 11) + 3a = 20a - 31$

$$4a - 44 + 3a = 20a - 31$$

$$7a - 44 = 20a - 31$$

$$-13a = 13$$

$$a = -1$$

The solution set is $\{-1\}$.

9. $2q - 1 = -7$

$$2q = -6$$

$$q = -3$$

The solution set is $\{-3\}$.

10. $|3q - 7| - 4 = 0$

$$|3q - 7| = 4$$

$$3q - 7 = 4 \quad \text{or} \quad 3q - 7 = -4$$

$$3q = 11$$

$$3q = 3$$

$$q = \frac{11}{3}$$

$$\text{or} \quad q = 1$$

The solution set is $\{1, \frac{11}{3}\}$.

$$\begin{aligned} 11. \quad 6z - 5 &\leq 3z + 10 \\ 3z &\leq 15 \\ z &\leq 5 \end{aligned}$$

The solution set is $(-\infty, 5]$.

$$\begin{aligned} 12. \quad |5z - 8| + 9 &\geq 7 \\ |5z - 8| &\geq -2 \end{aligned}$$

The absolute value of an expression is always nonnegative, so the inequality is true for any real number z .

The solution set is $(-\infty, \infty)$.

$$\begin{aligned} 13. \quad 9x - 3(x + 1) &= 8x - 7 \\ 9x - 3x - 3 &= 8x - 7 \\ 6x - 3 &= 8x - 7 \\ 4 &= 2x \\ 2 &= x \end{aligned}$$

The solution set is $\{2\}$.

$$\begin{aligned} 14. \quad |m| &\geq 8 \\ m &\geq 8 \quad \text{or} \quad m \leq -8 \end{aligned}$$

The solution set is $(-\infty, -8] \cup [8, \infty)$.

$$\begin{aligned} 15. \quad 9x - 5 &\geq 9x + 3 \\ -5 &\geq 3 \quad \text{False} \end{aligned}$$

This is a false statement, so the inequality is a contradiction.

The solution set is $\emptyset$.

$$\begin{aligned} 16. \quad 13p - 5 &> 13p - 8 \\ -5 &> -8 \end{aligned}$$

This inequality is true for every value of p .

The solution set is $(-\infty, \infty)$.

$$\begin{aligned} 17. \quad |q| &< 5.5 \\ -5.5 &< q < 5.5 \end{aligned}$$

The solution set is $(-5.5, 5.5)$.

$$\begin{aligned} 18. \quad 4z - 1 &= 12 + z \\ 3z &= 13 \\ z &= \frac{13}{3} \end{aligned}$$

The solution set is $\{\frac{13}{3}\}$.

$$\begin{aligned} 19. \quad \frac{2}{3}x + 8 &= \frac{1}{4}x \\ 8x + 96 &= 3x \quad \text{Multiply by 12.} \\ 5x &= -96 \\ x &= -\frac{96}{5} \end{aligned}$$

The solution set is $\{-\frac{96}{5}\}$.

$$\begin{aligned} 20. \quad -\frac{5}{8}z &\geq -20 \\ -\frac{8}{5}\left(-\frac{5}{8}z\right) &\leq -\frac{8}{5}(-20) \\ z &\leq 32 \end{aligned}$$

The solution set is $(-\infty, 32]$.

$$\begin{aligned} 21. \quad \frac{1}{4}p &< -6 \\ 4\left(\frac{1}{4}p\right) &< 4(-6) \\ p &< -24 \end{aligned}$$

The solution set is $(-\infty, -24)$.

$$\begin{aligned} 22. \quad 7z - 3 + 2z &= 9z - 8z \\ 9z - 3 &= z \\ 8z &= 3 \\ z &= \frac{3}{8} \end{aligned}$$

The solution set is $\{\frac{3}{8}\}$.

$$\begin{aligned} 23. \quad \frac{3}{5}q - \frac{1}{10} &= 2 \\ 6q - 1 &= 20 \quad \text{Multiply by 10.} \\ 6q &= 21 \\ q &= \frac{21}{6} = \frac{7}{2} \end{aligned}$$

The solution set is $\{\frac{7}{2}\}$.

$$\begin{aligned} 24. \quad |r - 1| &< 7 \\ -7 &< r - 1 < 7 \\ -6 &< r < 8 \end{aligned}$$

The solution set is $(-6, 8)$.

$$\begin{aligned} 25. \quad r + 9 + 7r &= 4(3 + 2r) - 3 \\ 8r + 9 &= 12 + 8r - 3 \\ 8r + 9 &= 8r + 9 \\ 0 &= 0 \quad \text{True} \end{aligned}$$

The last statement is true for any real number r .

The solution set is $(-\infty, \infty)$.

$$\begin{aligned} 26. \quad 6 - 3(2 - p) &< 2(1 + p) + 3 \\ 6 - 6 + 3p &< 2 + 2p + 3 \\ 3p &< 5 + 2p \\ p &< 5 \end{aligned}$$

The solution set is $(-\infty, 5)$.

$$\begin{aligned} 27. \quad |2p - 3| &> 11 \\ 2p - 3 &> 11 \quad \text{or} \quad 2p - 3 < -11 \\ 2p &> 14 \quad \quad \quad 2p < -8 \\ p &> 7 \quad \quad \quad p < -4 \end{aligned}$$

The solution set is $(-\infty, -4) \cup (7, \infty)$.

$$\begin{aligned} 28. \quad \frac{x}{4} - \frac{2x}{3} &= -10 \\ 3x - 8x &= -120 \quad \text{Multiply by 12.} \\ -5x &= -120 \\ x &= 24 \end{aligned}$$

The solution set is $\{24\}$.

$$\begin{aligned} 29. \quad |5a + 1| &\leq 0 \end{aligned}$$

The expression $|5a + 1|$ is never less than 0 since an absolute value expression must be nonnegative.

However, $|5a + 1| = 0$ if

$$\begin{aligned}5a + 1 &= 0 \\5a &= -1 \\a &= \frac{-1}{5} = -\frac{1}{5}\end{aligned}$$

The solution set is $\{-\frac{1}{5}\}$.

$$\begin{aligned}30. \quad 5z - (3 + z) &\geq 2(3z + 1) \\5z - 3 - z &\geq 6z + 2 \\4z - 3 &\geq 6z + 2 \\-2z &\geq 5 \\z &\leq -\frac{5}{2}\end{aligned}$$

The solution set is $(-\infty, -\frac{5}{2}]$.

$$\begin{aligned}31. \quad -2 &\leq 3x - 1 \leq 8 \\-1 &\leq 3x \leq 9 \\-\frac{1}{3} &\leq x \leq 3\end{aligned}$$

The solution set is $[-\frac{1}{3}, 3]$.

$$\begin{aligned}32. \quad -1 &\leq 6 - x \leq 5 \\-7 &\leq -x \leq -1 \\7 &\geq x \geq 1 \quad \text{or} \quad 1 \leq x \leq 7\end{aligned}$$

The solution set is $[1, 7]$.

$$\begin{aligned}33. \quad |7z - 1| &= |5z + 3| \\7z - 1 &= 5z + 3 \quad \text{or} \quad 7z - 1 = -(5z + 3) \\2z &= 4 \quad \quad \quad 7z - 1 = -5z - 3 \\12z &= -2 \\z &= 2 \quad \quad \quad \text{or} \quad z = \frac{-2}{12} = -\frac{1}{6}\end{aligned}$$

The solution set is $\{-\frac{1}{6}, 2\}$.

$$\begin{aligned}34. \quad |p + 2| &= |p + 4| \\p + 2 &= p + 4 \quad \text{or} \quad p + 2 = -(p + 4) \\2 &= 4 \quad \text{False} \quad \quad p + 2 = -p - 4 \\2p &= -6 \\p &= -3\end{aligned}$$

No solution or $p = -3$

The solution set is $\{-3\}$.

$$\begin{aligned}35. \quad |1 - 3x| &\geq 4 \\1 - 3x &\geq 4 \quad \text{or} \quad 1 - 3x \leq -4 \\-3x &\geq 3 \quad \quad \quad -3x \leq -5 \\x &\leq -1 \quad \text{or} \quad x \geq \frac{5}{3}\end{aligned}$$

The solution set is $(-\infty, -1] \cup [\frac{5}{3}, \infty)$.

$$\begin{aligned}36. \quad \frac{1}{2} &\leq \frac{2}{3}r \leq \frac{5}{4} \\6 &\leq 8r \leq 15 \quad \text{Multiply by 12.} \\ \frac{6}{8} &\leq r \leq \frac{15}{8} \\ \frac{3}{4} &\leq r \leq \frac{15}{8}\end{aligned}$$

The solution set is $[\frac{3}{4}, \frac{15}{8}]$.

$$\begin{aligned}37. \quad -(m + 4) + 2 &= 3m + 8 \\-m - 4 + 2 &= 3m + 8 \\-m - 2 &= 3m + 8 \\-10 &= 4m \\m &= \frac{-10}{4} = -\frac{5}{2}\end{aligned}$$

The solution set is $\{-\frac{5}{2}\}$.

$$\begin{aligned}38. \quad \frac{p}{6} - \frac{3p}{5} &= p - 86 \\5p - 18p &= 30p - 2580 \quad \text{Multiply by 30.} \\-43p &= -2580 \\p &= \frac{-2580}{-43} = 60\end{aligned}$$

The solution set is $\{60\}$.

$$\begin{aligned}39. \quad -6 &\leq \frac{3}{2} - x \leq 6 \\-12 &\leq 3 - 2x \leq 12 \\-15 &\leq -2x \leq 9 \\\frac{15}{2} &\geq x \geq -\frac{9}{2} \quad \text{or} \quad -\frac{9}{2} \leq x \leq \frac{15}{2}\end{aligned}$$

The solution set is $[-\frac{9}{2}, \frac{15}{2}]$.

$$\begin{aligned}40. \quad |5 - x| &< 4 \\-4 &< 5 - x < 4 \\-9 &< -x < -1 \\9 &> x > 1 \quad \text{or} \quad 1 < x < 9\end{aligned}$$

The solution set is $(1, 9)$.

$$\begin{aligned}41. \quad |x - 1| &\geq -6\end{aligned}$$

The absolute value of an expression is always nonnegative, so the inequality is true for any real number x .

The solution set is $(-\infty, \infty)$.

$$\begin{aligned}42. \quad |2r - 5| &= |r + 4| \\2r - 5 &= r + 4 \quad \text{or} \quad 2r - 5 = -(r + 4) \\2r - 5 &= r + 4 \quad \quad \quad 2r - 5 = -r - 4 \\3r &= 1 \\r &= 9 \quad \quad \quad \text{or} \quad r = \frac{1}{3}\end{aligned}$$

The solution set is $\{\frac{1}{3}, 9\}$.

$$\begin{aligned}43. \quad 8q - (1 - q) &= 3(1 + 3q) - 4 \\8q - 1 + q &= 3 + 9q - 4 \\9q - 1 &= 9q - 1 \quad \text{True}\end{aligned}$$

This is an identity.

The solution set is $(-\infty, \infty)$.

$$\begin{aligned}44. \quad 8t - (t + 3) &= -(2t + 1) - 12 \\8t - t - 3 &= -2t - 1 - 12 \\7t - 3 &= -2t - 13 \\9t &= -10 \\t &= -\frac{10}{9}\end{aligned}$$

The solution set is $\{-\frac{10}{9}\}$.

$$\begin{aligned}45. \quad |r - 5| &= |r + 9| \\r - 5 &= r + 9 \quad \text{or} \quad r - 5 = -(r + 9) \\-5 &= 9 \quad \text{False} \quad \quad r - 5 = -r - 9 \\2r &= -4 \\r &= -2\end{aligned}$$

The solution set is $\{-2\}$.

46. $|r + 2| < -3$

There are no numbers whose absolute value is negative, so this inequality has no solution.

The solution set is $\emptyset$.

47. $2x + 1 > 5$ or $3x + 4 < 1$

$2x > 4$ $3x < -3$

$x > 2$ or $x < -1$

The solution set is $(-\infty, -1) \cup (2, \infty)$.

48. $1 - 2x \geq 5$ and $7 + 3x \geq -2$

$-2x \geq 4$ $3x \geq -9$

$x \leq -2$ and $x \geq -3$

This is an intersection.

The solution set is $[-3, -2]$.

Chapter 2 Review Exercises

1. $-(8 + 3z) + 5 = 2z + 6$

$-8 - 3z + 5 = 2z + 6$

$-3z - 3 = 2z + 6$

$-5z = 9$

$z = -\frac{9}{5}$

The solution set is $\{-\frac{9}{5}\}$.

2. $-\frac{3}{4}x = -12$

$-3x = -48$ Multiply by 4.

$x = 16$

The solution set is $\{16\}$.

3. $\frac{2q + 1}{3} - \frac{q - 1}{4} = 0$

$4(2q + 1) - 3(q - 1) = 0$ Multiply by 12.

$8q + 4 - 3q + 3 = 0$

$5q + 7 = 0$

$5q = -7$

$q = -\frac{7}{5}$

The solution set is $\{-\frac{7}{5}\}$.

4. $5(2x - 3) = 6(x - 1) + 4x$

$10x - 15 = 6x - 6 + 4x$

$10x - 15 = 10x - 6$

$-15 = -6$ False

This is a false statement, so the equation is a contradiction.

The solution set is $\emptyset$.

5. $7r - 3(2r - 5) + 5 + 3r = 4r + 20$

$7r - 6r + 15 + 5 + 3r = 4r + 20$

$4r + 20 = 4r + 20$

$20 = 20$ True

This equation is an identity.

The solution set is $(-\infty, \infty)$.

6. $8p - 4p - (p - 7) + 9p + 6 = 12p - 7$

$8p - 4p - p + 7 + 9p + 6 = 12p - 7$

$12p + 13 = 12p - 7$

$13 = -7$ False

This equation is a contradiction.

The solution set is $\emptyset$.

7. $-2r + 6(r - 1) + 3r - (4 - r) = -(r + 5) - 5$

$-2r + 6r - 6 + 3r - 4 + r = -r - 5 - 5$

$8r - 10 = -r - 10$

$9r = 0$

$r = 0$

This equation is a conditional equation.

The solution set is $\{0\}$.

8. Solve $V = LWH$ for L .

$$\frac{V}{WH} = \frac{LWH}{WH}$$

$$\frac{V}{WH} = L \text{ or } L = \frac{V}{WH}$$

9. Solve $A = \frac{1}{2}h(b + B)$ for b .

$2A = h(b + B) \text{ or } 2A = hb + hB$

$\frac{2A}{h} = b + B \text{ or } 2A - hB = hb$

$\frac{2A}{h} - B = b \text{ or } \frac{2A - hB}{h} = b$

10. Solve $M = -\frac{1}{4}(x + 3y)$ for x .

$-4M = x + 3y$ Multiply by -4 .

$x = -4M - 3y$

11. Solve $P = \frac{3}{4}x - 12$ for x .

$P + 12 = \frac{3}{4}x$ Add 12.

$x = \frac{4}{3}(P + 12)$ Multiply by $4/3$.

$x = \frac{4}{3}P + 16$

12. Solve $-2x + 5 = 7$.

Begin by subtracting 5 from each side. Then divide each side by -2 .

13. Use the formula $V = LWH$ and substitute 180 for V , 6 for L , and 5 for W .

$180 = 6(5)H$

$180 = 30H$

$6 = H$

The height is 6 feet.

14. Divide the amount of decrease by the original amount.

$$\frac{17,849 - 15,798}{17,849} = \frac{2051}{17,849} \approx 0.115$$

The percent decrease is about 11.5%.

15. Use the formula $I = prt$. Substitute \$7800 for I , \$30,000 for p , and 4 for t . Solve for r .

$$\begin{aligned} I &= prt \\ \$7800 &= (\$30,000)r(4) \\ 7800 &= 120,000r \\ r &= \frac{7800}{120,000} = 0.065 \end{aligned}$$

The rate is 6.5%.

16. Use the formula $C = \frac{5}{9}(F - 32)$ and substitute 77 for F .

$$C = \frac{5}{9}(77 - 32) = \frac{5}{9}(45) = 25$$

The Celsius temperature is 25°.

17. The amount of money spent on Social Security in 2005 was about

$$0.21(\$2500 \text{ billion}) = \$525 \text{ billion.}$$

18. The amount of money spent on education and social services in 2005 was about

$$0.039(\$2500 \text{ billion}) = \$97.5 \text{ billion}$$

19. "One-third of a number, subtracted from 9" is written

$$9 - \frac{1}{3}x.$$

20. "The product of 4 and a number, divided by 9 more than the number" is written

$$\frac{4x}{x + 9}.$$

21. Let x = the width of the rectangle. Then $2x - 3$ = the length of the rectangle.

Use the formula $P = 2L + 2W$ with $P = 42$.

$$\begin{aligned} 42 &= 2(2x - 3) + 2x \\ 42 &= 4x - 6 + 2x \\ 48 &= 6x \\ 8 &= x \end{aligned}$$

The width is 8 meters and the length is $2(8) - 3 = 13$ meters.

22. Let x = the length of each equal side. Then $2x - 15$ = the length of the third side.

Use the formula $P = a + b + c$ with $P = 53$.

$$\begin{aligned} 53 &= x + x + (2x - 15) \\ 53 &= 4x - 15 \\ 68 &= 4x \\ 17 &= x \end{aligned}$$

The lengths of the three sides are 17 inches, 17 inches, and $2(17) - 15 = 19$ inches.

23. Let x = the number of kilograms of peanut clusters. Then $3x$ is the number of kilograms of chocolate creams.

The clerk has a total of 48 kg.

$$\begin{aligned} x + 3x &= 48 \\ 4x &= 48 \\ x &= 12 \end{aligned}$$

The clerk has 12 kilograms of peanut clusters.

24. Let x = the number of liters of the 20% solution. Make a table.

Liters of Solution	Percent (as a decimal)	Liters of Pure Chemical
x	0.20	$0.20x$
15	0.50	$0.50(15) = 7.5$
$x + 15$	0.30	$0.30(x + 15)$

The last column gives the equation.

$$\begin{aligned} 0.20x + 7.5 &= 0.30(x + 15) \\ 0.20x + 7.5 &= 0.30x + 4.5 \\ 3 &= 0.10x \\ 30 &= x \end{aligned}$$

30 L of the 20% solution should be used.

25.

Liters of Solution	Percent (as a decimal)	Liters of Pure Acid
30	0.40	$0.40(30) = 12$
x	0	$0(x) = 0$
$30 + x$	0.30	$0.30(30 + x)$

The last column gives the equation.

$$\begin{aligned} 12 + 0 &= 0.30(30 + x) \\ 12 &= 9 + 0.3x \\ 3 &= 0.3x \\ 10 &= x \end{aligned}$$

10 L of water should be added.

26. Let x = the amount invested at 6%. Then $x - 4000$ = the amount invested at 4%.

Principal	Rate (as a decimal)	Interest
x	0.06	$0.06x$
$x - 4000$	0.04	$0.04(x - 4000)$
	Total →	\$840

The last column gives the equation.

$$\begin{aligned} 0.06x + 0.04(x - 4000) &= 840 \\ 6x + 4(x - 4000) &= 84,000 && \text{Multiply by 100.} \\ 6x + 4x - 16,000 &= 84,000 \\ 10x &= 100,000 \\ x &= 10,000 \end{aligned}$$

Jay should invest \$10,000 at 6% and \$10,000 - \$4000 = \$6000 at 4%.

27. Let x = the number of quarters. Then $2x - 1$ is the number of dimes.

Number of Coins	Denomination	Value
x	0.25	$0.25x$
$2x - 1$	0.10	$0.10(2x - 1)$
	Total →	3.50

The sum of the values equals the total value.

$$0.25x + 0.10(2x - 1) = 3.50$$

Multiply by 100.

$$25x + 10(2x - 1) = 350$$

$$25x + 20x - 10 = 350$$

$$45x = 360$$

$$x = 8$$

There are 8 quarters and $2(8) - 1 = 15$ dimes.

Check $8(0.25) + 15(0.10) = 3.50$

28. Let x = the number of nickels.
Then $19 - x$ is the number of dimes.

Number of Coins	Denomination	Value
x	0.05	$0.05x$
$19 - x$	0.10	$0.10(19 - x)$
	Total →	1.55

The sum of the values equals the total value.

$$0.05x + 0.10(19 - x) = 1.55$$

Multiply by 100.

$$5x + 10(19 - x) = 155$$

$$5x + 190 - 10x = 155$$

$$-5x = -35$$

$$x = 7$$

He had 7 nickels and $19 - 7 = 12$ dimes.

Check $7(0.05) + 12(0.10) = 1.55$

29. Use the formula $d = rt$ or $r = \frac{d}{t}$. Here, d is about 400 mi and t is about 8 hr. Since $\frac{400}{8} = 50$, the best estimate is choice **A**.

30. Use the formula $d = rt$.

(a) Here, $r = 53$ mph and $t = 10$ hr.

$$d = 53(10) = 530$$

The distance is 530 miles.

(b) Here, $r = 164$ mph and $t = 2$ hr.

$$d = 164(2) = 328$$

The distance is 328 miles.

31. Let x = the time it takes for the trains to be 297 mi apart.

Use the formula $d = rt$.

	Rate	Time	Distance
Passenger Train	60	x	$60x$
Freight Train	75	x	$75x$
			297

The total distance traveled is the sum of the distances traveled by each train.

$$60x + 75x = 297$$

$$135x = 297$$

$$x = 2.2$$

It will take the trains 2.2 hours before they are 297 miles apart.

32. Let x = the speed of the faster car and
 $x - 15$ = the speed of the slower car.
Make a table.

	Rate	Time	Distance
Faster Car	x	2	$2x$
Slower Car	$x - 15$	2	$2(x - 15)$
			230

The total distance traveled is the sum of the distances traveled by each car.

$$2x + 2(x - 15) = 230$$

$$2x + 2x - 30 = 230$$

$$4x = 260$$

$$x = 65$$

The faster car travels at 65 km/hr, while the slower car travels at $65 - 15 = 50$ km/hr.

Check $2(65) + 2(50) = 230$

33. Let x = amount of time spent averaging 45 miles per hour. Then $4 - x$ = amount of time at 50 mph.

	Rate	Time	Distance
First Part	45	x	$45x$
Second Part	50	$4 - x$	$50(4 - x)$
Total			195

From the last column:

$$45x + 50(4 - x) = 195$$

$$45x + 200 - 50x = 195$$

$$-5x = -5$$

$$x = 1$$

The automobile averaged 45 mph for 1 hour.

Check 45 mph for 1 hour = 45 miles and 50 mph for 3 hours = 150 miles; $45 + 150 = 195$.

34. Let x = the average speed for the first hour. Then $x - 7$ = the average speed for the second hour. Using $d = rt$, the distance traveled for the first hour is $x(1)$, for the second hour is $(x - 7)(1)$, and for the whole trip, 85.

$$\begin{aligned}x + (x - 7) &= 85 \\2x - 7 &= 85 \\2x &= 92 \\x &= 46\end{aligned}$$

The average speed for the first hour was 46 mph.

Check 46 mph for 1 hour = 46 miles and $46 - 7 = 39$ mph for 1 hour = 39 miles; $46 + 39 = 85$.

35. The sum of the angles in a triangle is 180° .

$$\begin{aligned}(3x + 7) + (4x + 1) + (9x - 4) &= 180 \\16x + 4 &= 180 \\16x &= 176 \\x &= 11\end{aligned}$$

The first angle is $3(11) + 7 = 40^\circ$.

The second angle is $4(11) + 1 = 45^\circ$.

The third angle is $9(11) - 4 = 95^\circ$.

36. The marked angles are supplements which have a sum of 180° .

$$\begin{aligned}(15x + 15) + (3x + 3) &= 180 \\18x + 18 &= 180 \\18x &= 162 \\x &= 9\end{aligned}$$

The angle measures are

$15(9) + 15 = 150^\circ$ and $3(9) + 3 = 30^\circ$.

37. $-\frac{2}{3}k < 6$

$$-2k < 18 \quad \text{Multiply by 3.}$$

Divide by -2 ; reverse the inequality symbol.

$$k > -9$$

The solution set is $(-9, \infty)$.

38. $-5x - 4 \geq 11$

$$-5x \geq 15$$

Divide by -5 ; reverse the inequality symbol.

$$x \leq -3$$

The solution set is $(-\infty, -3]$.

39. $\frac{6a + 3}{-4} < -3$

Multiply by -4 ; reverse the inequality symbol.

$$6a + 3 > 12$$

$$6a > 9$$

$$a > \frac{9}{6} = \frac{3}{2}$$

The solution set is $(\frac{3}{2}, \infty)$.

40. $5 - (6 - 4k) \geq 2k - 7$
 $5 - 6 + 4k \geq 2k - 7$
 $4k - 1 \geq 2k - 7$
 $2k \geq -6$
 $k \geq -3$

The solution set is $[-3, \infty)$.

41. $8 \leq 3z - 1 < 14$

$$9 \leq 3z < 15$$

$$3 \leq z < 5$$

The solution set is $[3, 5)$.

42. $\frac{5}{3}(m - 2) + \frac{2}{5}(m + 1) > 1$
 $25(m - 2) + 6(m + 1) > 15$

Multiply by 15.

$$25m - 50 + 6m + 6 > 15$$

$$31m - 44 > 15$$

$$31m > 59$$

$$m > \frac{59}{31}$$

The solution set is $(\frac{59}{31}, \infty)$.

43. Let x = the other dimension of the rectangle. One dimension of the rectangle is 22 and the perimeter can be no greater than 120.

$$\begin{aligned}P &\leq 120 \\2L + 2W &\leq 120 \\2(x) + 2(22) &\leq 120 \\2x + 44 &\leq 120 \\2x &\leq 76 \\x &\leq 38\end{aligned}$$

The other dimension must be 38 meters or less.

44. Let x = the number of tickets that can be purchased. The total cost of the tickets is $\$89x$. Including the $\$50$ discount and staying within the available $\$2000$, we have

$$89x - 50 \leq 2000.$$

$$89x \leq 2050$$

$$x \lesssim 23.03$$

The group can purchase 23 tickets or less.

45. Let x = the student's score on the fifth test. The average of the five test scores must be at least 70. The inequality is

$$\frac{75 + 79 + 64 + 71 + x}{5} \geq 70.$$

$$75 + 79 + 64 + 71 + x \geq 350$$

$$289 + x \geq 350$$

$$x \geq 61$$

The student will pass algebra if any score greater than or equal to 61% on the fifth test is achieved.

46. The result, $-8 < -13$, is a false statement. There are no real numbers that make this inequality true. The solution set is $\emptyset$.

For Exercises 47–50, let $A = \{a, b, c, d\}$,
 $B = \{a, c, e, f\}$, and $C = \{a, e, f, g\}$.

47. $A \cap B = \{a, b, c, d\} \cap \{a, c, e, f\}$
 $= \{a, c\}$

48. $A \cap C = \{a, b, c, d\} \cap \{a, e, f, g\}$
 $= \{a\}$

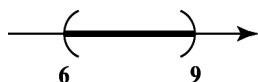
49. $B \cup C = \{a, c, e, f\} \cup \{a, e, f, g\}$
 $= \{a, c, e, f, g\}$

50. $A \cup C = \{a, b, c, d\} \cup \{a, e, f, g\}$
 $= \{a, b, c, d, e, f, g\}$

51. $x > 6$ and $x < 9$

The graph of the solution set will be all numbers which are both greater than 6 and less than 9. The overlap is the numbers between 6 and 9, not including the endpoints.

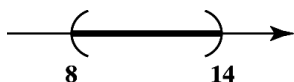
The solution set is $(6, 9)$.



52. $x + 4 > 12$ and $x - 2 < 12$
 $x > 8$ and $x < 14$

The graph of the solution set will be all numbers between 8 and 14, not including the endpoints.

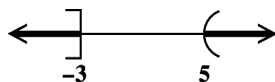
The solution set is $(8, 14)$.



53. $x > 5$ or $x \leq -3$

The graph of the solution set will be all numbers that are either greater than 5 or less than or equal to -3 .

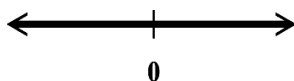
The solution set is $(-\infty, -3] \cup (5, \infty)$.



54. $x \geq -2$ or $x < 2$

The graph of the solution set will be all numbers that are either greater than or equal to -2 or less than 2. All real numbers satisfy these criteria.

The solution set is $(-\infty, \infty)$.



55. $x - 4 > 6$ and $x + 3 \leq 10$
 $x > 10$ and $x \leq 7$

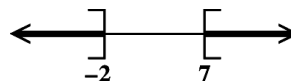
The graph of the solution set will be all numbers that are both greater than 10 and less than or equal to 7. There are no real numbers satisfying these criteria.

The solution set is $\emptyset$.

56. $-5x + 1 \geq 11$ or $3x + 5 \geq 26$
 $-5x \geq 10$ or $3x \geq 21$
 $x \leq -2$ or $x \geq 7$

The graph of the solution set will be all numbers that are either less than or equal to -2 or greater than or equal to 7.

The solution set is $(-\infty, -2] \cup [7, \infty)$.



57. $(-3, \infty) \cap (-\infty, 4)$
 $(-3, \infty)$ includes all real numbers greater than -3 .

$(-\infty, 4)$ includes all real numbers less than 4.
Find the intersection. The numbers common to both sets are greater than -3 and less than 4.

$$-3 < x < 4$$

The solution set is $(-3, 4)$.

58. $(-\infty, 6) \cap (-\infty, 2)$
 $(-\infty, 6)$ includes all real numbers less than 6.
 $(-\infty, 2)$ includes all real numbers less than 2.
Find the intersection. The numbers common to both sets are less than 2.

The solution set is $(-\infty, 2)$.

59. $(4, \infty) \cup (9, \infty)$
 $(4, \infty)$ includes all real numbers greater than 4.
 $(9, \infty)$ includes all real numbers greater than 9.
Find the union. The numbers in the first set, the second set, or in both sets are all the real numbers that are greater than 4.

The solution set is $(4, \infty)$.

60. $(1, 2) \cup (1, \infty)$
 $(1, 2)$ includes the real numbers between 1 and 2, not including 1 and 2.
 $(1, \infty)$ includes all real numbers greater than 1.
Find the union. The numbers in the first set, the second set, or in both sets are all real numbers greater than 1.

The solution set is $(1, \infty)$.

61. $|x| = 7$
 $x = 7$ or $x = -7$
The solution set is $\{-7, 7\}$.

62. $|x + 2| = 9$
 $x + 2 = 9$ or $x + 2 = -9$
 $x = 7$ or $x = -11$

The solution set is $\{-11, 7\}$.

63. $|3k - 7| = 8$

$3k - 7 = 8 \quad \text{or} \quad 3k - 7 = -8$

$3k = 15 \quad 3k = -1$

$k = 5 \quad \text{or} \quad k = -\frac{1}{3}$

The solution set is $\{-\frac{1}{3}, 5\}$.

64. $|z - 4| = -12$

Since the absolute value of an expression can never be negative, there are no solutions for this equation.

The solution set is $\emptyset$.

65. $|2k - 7| + 4 = 11$

$|2k - 7| = 7$

$2k - 7 = 7 \quad \text{or} \quad 2k - 7 = -7$

$2k = 14 \quad 2k = 0$

$k = 7 \quad \text{or} \quad k = 0$

The solution set is $\{0, 7\}$.

66. $|4a + 2| - 7 = -3$

$|4a + 2| = 4$

$4a + 2 = 4 \quad \text{or} \quad 4a + 2 = -4$

$4a = 2 \quad 4a = -6$

$a = \frac{2}{4} \quad a = -\frac{6}{4}$

$a = \frac{1}{2} \quad \text{or} \quad a = -\frac{3}{2}$

The solution set is $\{-\frac{3}{2}, \frac{1}{2}\}$.

67. $|3p + 1| = |p + 2|$

$3p + 1 = p + 2 \quad \text{or} \quad 3p + 1 = -(p + 2)$

$2p = 1 \quad 3p + 1 = -p - 2$

$4p = -3$

$p = \frac{1}{2} \quad \text{or} \quad p = -\frac{3}{4}$

The solution set is $\{-\frac{3}{4}, \frac{1}{2}\}$.

68. $|2m - 1| = |2m + 3|$

$2m - 1 = 2m + 3 \quad \text{or} \quad 2m - 1 = -(2m + 3)$

$-1 = 3 \quad \text{False} \quad 2m - 1 = -2m - 3$

$4m = -2$

$\text{No solution} \quad \text{or} \quad m = -\frac{2}{4} = -\frac{1}{2}$

The solution set is $\{-\frac{1}{2}\}$.

69. $|p| < 14$

$-14 < p < 14$

The solution set is $(-14, 14)$.

70. $|-t + 6| \leq 7$

$-7 \leq -t + 6 \leq 7$

$-13 \leq -t \leq 1$

Multiply by -1 ; reverse the inequality symbols.

$13 \geq t \geq -1 \quad \text{or} \quad -1 \leq t \leq 13$

The solution set is $[-1, 13]$.

71. $|2p + 5| \leq 1$

$-1 \leq 2p + 5 \leq 1$

$-6 \leq 2p \leq -4$

$-3 \leq p \leq -2$

The solution set is $[-3, -2]$.

72. $|x + 1| \geq -3$

Since the absolute value of an expression is always nonnegative (positive or zero), the inequality is *true* for any real number x .

The solution set is $(-\infty, \infty)$.

73. $5 - (6 - 4k) > 2k - 5$

$5 - 6 + 4k > 2k - 5$

$-1 + 4k > 2k - 5$

$2k > -4$

$k > -2$

The solution set is $(-2, \infty)$.

74. Solve $ak + bt = 6t - sk$ for k .

Get the " k -terms" on one side.

$ak + sk = 6t - bt$

$(a + s)k = 6t - bt$

$k = \frac{6t - bt}{a + s}$

If we take the k -terms to the right side in the first step, we get

$k = \frac{bt - 6t}{-a - s}.$

75. $x < 3$ and $x \geq -2$

The real numbers that are common to both sets are the numbers greater than or equal to -2 and less than 3 .

$-2 \leq x < 3$

The solution set is $[-2, 3)$.

76. $\frac{4x + 2}{4} + \frac{3x - 1}{8} = \frac{x + 6}{16}$

Clear fractions by multiplying by the LCD, 16.

$4(4x + 2) + 2(3x - 1) = x + 6$

$16x + 8 + 6x - 2 = x + 6$

$22x + 6 = x + 6$

$21x = 0$

$x = 0$

The solution set is $\{0\}$.

77. $|3k + 6| \geq 0$

The absolute value of an expression is always nonnegative, so the inequality is true for any real number k .

The solution set is $(-\infty, \infty)$.

88 Chapter 2 Linear Equations, Inequalities, and Applications

78. $-5r \geq -10$
 $r \leq 2$ Divide by -5 ; reverse symbol.

The solution set is $(-\infty, 2]$.

79. Use the formula $V = LWH$, and solve for H .

$$\frac{V}{LW} = \frac{LWH}{LW}$$

$$\frac{V}{LW} = H \quad \text{or} \quad H = \frac{V}{LW}$$

Substitute 1.5 for W , 5 for L , and 75 for V .

$$H = \frac{75}{5(1.5)} = \frac{75}{7.5} = 10$$

The height of the box is 10 ft.

80. Let x = the first consecutive integer. Then
 $x + 1$ = the second consecutive integer and
 $x + 2$ = the third consecutive integer.
 The sum of the first and third integers is 47 more than the second integer, so an equation is

$$x + (x + 2) = 47 + (x + 1).$$

$$2x + 2 = 48 + x$$

$$x = 46$$

Then $x + 1 = 47$, and $x + 2 = 48$.

The integers are 46, 47, and 48.

81. $|3x + 2| + 4 = 9$
 $|3x + 2| = 5$

$$3x + 2 = 5 \quad \text{or} \quad 3x + 2 = -5$$

$$3x = 3 \quad \quad \quad 3x = -7$$

$$x = 1 \quad \text{or} \quad x = -\frac{7}{3}$$

The solution set is $\{-\frac{7}{3}, 1\}$.

82. $0.05x + 0.03(1200 - x) = 42$
 $5x + 3(1200 - x) = 4200$ Multiply by 100.
 $5x + 3600 - 3x = 4200$
 $2x = 600$
 $x = 300$

The solution set is $\{300\}$.

83. $|m + 3| \leq 13$
 $-13 \leq m + 3 \leq 13$
 $-16 \leq m \leq 10$

The solution set is $[-16, 10]$.

84. $\frac{3}{4}(a - 2) - \frac{1}{3}(5 - 2a) < -2$
 $9(a - 2) - 4(5 - 2a) < -24$
 Multiply by 12.
 $9a - 18 - 20 + 8a < -24$
 $17a - 38 < -24$
 $17a < 14$
 $a < \frac{14}{17}$

The solution set is $(-\infty, \frac{14}{17})$.

85. $-4 < 3 - 2k < 9$
 $-7 < -2k < 6$
 Divide by -2 ; reverse the inequality symbols.
 $\frac{7}{2} > k > -3$ or $-3 < k < \frac{7}{2}$

The solution set is $(-3, \frac{7}{2})$.

86. $-0.3x + 2.1(x - 4) \leq -6.6$
 $-3x + 21(x - 4) \leq -66$
 Multiply by 10.
 $-3x + 21x - 84 \leq -66$
 $18x - 84 \leq -66$
 $18x \leq 18$
 $x \leq 1$

The solution set is $(-\infty, 1]$.

87. Let x = the angle. Then $90 - x$ is its complement and $180 - x$ is its supplement.
 The complement of an angle measures 10° less than one-fifth of its supplement.

$$90 - x = \frac{1}{5}(180 - x) - 10$$

$$450 - 5x = 180 - x - 50 \quad \text{Multiply by 5.}$$

$$450 - 5x = 130 - x$$

$$320 = 4x$$

$$80 = x$$

The measure of the angle is 80° .

88. Let x = the employee's earnings during the fifth month. The average of the five months must be at least \$1000.

$$\frac{900 + 1200 + 1040 + 760 + x}{5} \geq 1000$$

$$900 + 1200 + 1040 + 760 + x \geq 5000$$

$$3900 + x \geq 5000$$

$$x \geq 1100$$

Any amount greater than or equal to \$1100 will qualify the employee for the pension plan.

89. $|5r - 1| > 14$
 $5r - 1 > 14$ or $5r - 1 < -14$
 $5r > 15$ $5r < -13$
 $r > 3$ or $r < -\frac{13}{5}$

The solution set is $(-\infty, -\frac{13}{5}) \cup (3, \infty)$.

90. $x \geq -2$ or $x < 4$

The solution set includes all numbers either greater than or equal to -2 or all numbers less than 4. This is the union and is the set of all real numbers. The solution set is $(-\infty, \infty)$.

91. Let x = the number of liters of the 20% solution. Then $x + 10$ is the number of liters of the resulting 40% solution.

Liters of Solution	Percent (as a decimal)	Liters of Mixture
x	0.20	$0.20x$
10	0.50	$0.50(10) = 5$
$x + 10$	0.40	$0.40(x + 10)$

From the last column:

$$0.20x + 5 = 0.40(x + 10)$$

$$0.20x + 5 = 0.40x + 4$$

$$1 = 0.20x$$

$$5 = x$$

5 L of the 20% solution should be used.

92. $|m - 1| = |2m + 3|$
 $m - 1 = 2m + 3$ or $m - 1 = -(2m + 3)$
 $m - 1 = -2m - 3$
 $3m = -2$
 $-4 = m$ or $m = -\frac{2}{3}$

The solution set is $\{-4, -\frac{2}{3}\}$.

93. $\frac{3x}{5} - \frac{x}{2} = 3$
 $6x - 5x = 30$ Multiply by 10.
 $x = 30$

The solution set is $\{30\}$.

94. $|m + 3| \leq 1$
 $-1 \leq m + 3 \leq 1$
 $-4 \leq m \leq -2$

The solution set is $[-4, -2]$.

95. $|3k - 7| = 4$
 $3k - 7 = 4$ or $3k - 7 = -4$
 $3k = 11$ or $3k = 3$
 $k = \frac{11}{3}$ or $k = 1$

The solution set is $\{1, \frac{11}{3}\}$.

96. $5(2x - 7) = 2(5x + 3)$
 $10x - 35 = 10x + 6$
 $-35 = 6$ False

This equation is a *contradiction*.

The solution set is $\emptyset$.

97. (a) $|5x + 3| < k$

If $k < 0$, then $|5x + 3|$ would be less than a negative number. Since the absolute value of an expression is always nonnegative (positive or zero), the solution set is $\emptyset$.

- (b) $|5x + 3| > k$

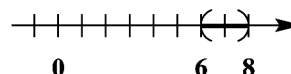
If $k < 0$, then $|5x + 3|$ would be greater than a negative number. Since the absolute value of an expression is always nonnegative (positive or zero), the solution set is the set of all real numbers, $(-\infty, \infty)$.

- (c) $|5x + 3| = k$

If $k < 0$, then $|5x + 3|$ would be equal to a negative number. Since the absolute value of an expression is always nonnegative (positive or zero), the solution set is $\emptyset$.

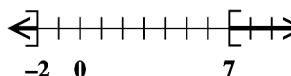
98. $x > 6$ and $x < 8$

The graph of the solution set is all numbers both greater than 6 *and* less than 8. This is the intersection. The elements common to both sets are the numbers between 6 and 8, not including the endpoints. The solution set is $(6, 8)$.



99. $-5x + 1 \geq 11$ or $3x + 5 \geq 26$
 $-5x \geq 10$ or $3x \geq 21$
 $x \leq -2$ or $x \geq 7$

The graph of the solution set is all numbers either less than or equal to -2 *or* greater than or equal to 7. This is the union. The solution set is $(-\infty, -2] \cup [7, \infty)$.



100. (a) The set of states with less than 1 million female workers is $\{\text{Maine, Oregon, Utah}\}$. The set of states with more than 1 million male workers is $\{\text{Illinois, North Carolina, Oregon, Wisconsin}\}$. Oregon is the only state in both sets, so the set of states with less than 1 million female workers *and* more than 1 million male workers is $\{\text{Oregon}\}$.

(b) The set of states with less than 1 million female workers *or* more than 2 million male workers is $\{\text{Illinois, Maine, North Carolina, Oregon, Utah}\}$.

(c) It is easy to see that the sum of the female and male workers for each state doesn't exceed 7 million, so the set of states with a total of more than 7 million civilian workers is $\{\}$, or $\emptyset$.

Chapter 2 Test

$$\begin{aligned}
 1. \quad & 3(2x - 2) - 4(x + 6) = 3x + 8 + x \\
 & 6x - 6 - 4x - 24 = 4x + 8 \\
 & 2x - 30 = 4x + 8 \\
 & -2x = 38 \\
 & x = -19
 \end{aligned}$$

The solution set is $\{-19\}$.

$$\begin{aligned}
 2. \quad & 0.08x + 0.06(x + 9) = 1.24 \\
 & 8x + 6(x + 9) = 124 \\
 & \quad \text{Multiply by 100.} \\
 & 8x + 6x + 54 = 124 \\
 & 14x + 54 = 124 \\
 & 14x = 70 \\
 & x = 5
 \end{aligned}$$

The solution set is $\{5\}$.

$$\begin{aligned}
 3. \quad & \frac{x+6}{10} + \frac{x-4}{15} = \frac{x+2}{6} \\
 & 3(x+6) + 2(x-4) = 5(x+2) \\
 & \quad \text{Multiply by 30.} \\
 & 3x + 18 + 2x - 8 = 5x + 10 \\
 & 5x + 10 = 5x + 10 \quad \text{True}
 \end{aligned}$$

This is an *identity*.

The solution set is $(-\infty, \infty)$.

$$\begin{aligned}
 4. \quad (a) \quad & 3x - (2 - x) + 4x + 2 = 8x + 3 \\
 & 3x - 2 + x + 4x + 2 = 8x + 3 \\
 & 8x = 8x + 3 \\
 & 0 = 3 \quad \text{False}
 \end{aligned}$$

The false statement indicates that the equation is a *contradiction*.

The solution set is $\emptyset$.

$$\begin{aligned}
 (b) \quad & \frac{x}{3} + 7 = \frac{5x}{6} - 2 - \frac{x}{2} + 9 \\
 & \text{Multiply each side by the LCD, 6.} \\
 & 2x + 42 = 5x - 12 - 3x + 54 \\
 & 2x + 42 = 2x + 42 \\
 & 0 = 0 \quad \text{True}
 \end{aligned}$$

This equation is an *identity*.

The solution set is $\{\text{all real numbers}\}$.

$$\begin{aligned}
 (c) \quad & -4(2x - 6) = 5x + 24 - 7x \\
 & -8x + 24 = -2x + 24 \\
 & 24 = 6x + 24 \\
 & 0 = 6x \\
 & 0 = x
 \end{aligned}$$

This is a *conditional equation*.

The solution set is $\{0\}$.

$$5. \quad \text{Solve } -16t^2 + vt - S = 0 \text{ for } v.$$

$$\begin{aligned}
 vt &= S + 16t^2 \\
 v &= \frac{S + 16t^2}{t}
 \end{aligned}$$

$$6. \quad \text{Solve } ar + 2 = 3r - 6t \text{ for } r. \\ \text{Get the "r-terms" on one side.}$$

$$\begin{aligned}
 ar - 3r &= -2 - 6t \\
 r(a - 3) &= -2 - 6t \\
 r &= \frac{-2 - 6t}{a - 3}
 \end{aligned}$$

If we take the r -terms to the right side in the first step, we get

$$r = \frac{2 + 6t}{3 - a}.$$

$$7. \quad \text{Solve } d = rt \text{ for } t \text{ and substitute 500 for } d \text{ and } 157.603 \text{ for } r.$$

$$t = \frac{d}{r} = \frac{500}{157.603} \approx 3.173$$

Wheldon's time was about 3.173 hr.

$$8. \quad \text{Use } I = Prt \text{ and substitute \$2281.25 for } I, \text{ \$36,500 for } P, \text{ and 1 for } t.$$

$$\begin{aligned}
 2281.25 &= 36,500r(1) \\
 r &= \frac{2281.25}{36,500} = 0.0625
 \end{aligned}$$

The rate of interest is 6.25%.

$$9. \quad \frac{27,385}{37,142} \approx 0.737$$

73.7% were classified as post offices.

$$10. \quad \text{Let } x = \text{the amount invested at 3\%. Then } 28,000 - x = \text{the amount invested at 5\%.}$$

Principal	Rate (as a decimal)	Interest
x	0.03	$0.03x$
$28,000 - x$	0.05	$0.05(28,000 - x)$
\$28,000	← Totals →	\$1240

From the last column:

$$\begin{aligned}
 0.03x + 0.05(28,000 - x) &= 1240 \\
 3x + 5(28,000 - x) &= 124,000 \\
 &\text{Multiply by 100.} \\
 3x + 140,000 - 5x &= 124,000 \\
 -2x &= -16,000 \\
 x &= 8000
 \end{aligned}$$

He invested \$8000 at 3% and
 $\$28,000 - \$8000 = \$20,000$ at 5%.

11. Let x = the speed of the slower car. Then $x + 15$ = the speed of the faster car.

Use the formula $d = rt$.

	Rate	Time	Distance
Slower Car	x	6	$6x$
Faster Car	$x + 15$	6	$6(x + 15)$
			630

The total distance traveled is the sum of the distances traveled by each car.

$$6x + 6(x + 15) = 630$$

$$6x + 6x + 90 = 630$$

$$12x = 540$$

$$x = 45$$

The slower car traveled at 45 mph, while the faster car traveled at $45 + 15 = 60$ mph.

12. The sum of the three angle measures is 180° .

$$(2x + 20) + x + x = 180$$

$$4x + 20 = 180$$

$$4x = 160$$

$$x = 40$$

The three angle measures are 40° , 40° , and $(2 \cdot 40 + 20)^\circ = 100^\circ$.

13. $4 - 6(x + 3) \leq -2 - 3(x + 6) + 3x$

$$4 - 6x - 18 \leq -2 - 3x - 18 + 3x$$

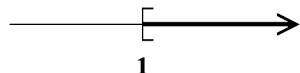
$$-6x - 14 \leq -20$$

$$-6x \leq -6$$

Divide by -6 , and reverse the inequality symbol.

$$x \geq 1$$

The solution set is $[1, \infty)$.



14. $-\frac{4}{7}x > -16$

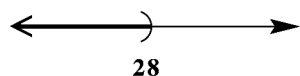
$$-4x > -112$$

Multiply by 7.

Divide by -4 , and reverse the inequality symbol.

$$x < 28$$

The solution set is $(-\infty, 28)$.



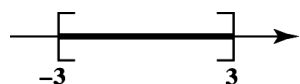
15. $-6 \leq \frac{4}{3}x - 2 \leq 2$

$$-18 \leq 4x - 6 \leq 6 \quad \text{Multiply by 3.}$$

$$-12 \leq 4x \leq 12 \quad \text{Add 6.}$$

$$-3 \leq x \leq 3 \quad \text{Divide by 4.}$$

The solution set is $[-3, 3]$.



16. For each inequality, divide both sides by -3 and reverse the direction of the inequality symbol.

$$\text{A. } -3x < 9 \quad \text{B. } -3x > -9$$

$$x > -3 \quad x < 3$$

$$\text{C. } -3x > 9 \quad \text{D. } -3x < -9$$

$$x < -3 \quad x > 3$$

Thus, inequality C is equivalent to $x < -3$.

17. Let x = the score on the fourth test.

$$\frac{83 + 76 + 79 + x}{4} \geq 80$$

$$\frac{238 + x}{4} \geq 80$$

$$238 + x \geq 320$$

$$x \geq 82$$

The minimum score must be 82 to guarantee a B.

18. $C = 50x + 5000$; $R = 60x$

$$R \geq C$$

$$60x \geq 50x + 5000$$

$$10x \geq 5000$$

$$x \geq 500$$

The solution set is $[500, \infty)$.

19. (a) $A \cap B = \{1, 2, 5, 7\} \cap \{1, 5, 9, 12\}$
 $= \{1, 5\}$

(b) $A \cup B = \{1, 2, 5, 7\} \cup \{1, 5, 9, 12\}$
 $= \{1, 2, 5, 7, 9, 12\}$

20. $3k \geq 6$ and $k - 4 < 5$
 $k \geq 2$ and $k < 9$

The solution set is all numbers both greater than or equal to 2 and less than 9. This is the intersection. The numbers common to both sets are between 2 and 9, including 2 but not 9. The solution set is $[2, 9)$.

21. $-4x \leq -24$ or $4x - 2 < 10$
 $4x < 12$
 $x \geq 6$ or $x < 3$

The solution set is all numbers less than 3 or greater than or equal to 6. This is the union. The solution set is $(-\infty, 3) \cup [6, \infty)$.

22. $|4x + 3| \leq 7$
 $-7 \leq 4x + 3 \leq 7$
 $-10 \leq 4x \leq 4$
 $-\frac{10}{4} \leq x \leq \frac{4}{4}$
 $-\frac{5}{2} \leq x \leq 1$

The solution set is $[-\frac{5}{2}, 1]$.

23. $|5 - 6x| > 12$

$$\begin{array}{lcl} 5 - 6x > 12 & \text{or} & 5 - 6x < -12 \\ -6x > 7 & & -6x < -17 \\ x < -\frac{7}{6} & \text{or} & x > \frac{17}{6} \end{array}$$

The solution set is $(-\infty, -\frac{7}{6}) \cup (\frac{17}{6}, \infty)$.

24. $|7 - x| \leq -1$

Since the absolute value of an expression is always nonnegative (positive or zero), the inequality is *false* for any real number x .

The solution set is $\emptyset$.

25. $|-3x + 4| - 4 < -1$

$$|-3x + 4| < 3$$

$$-3 < -3x + 4 < 3$$

$$-7 < -3x < -1$$

$$\frac{7}{3} > x > \frac{1}{3}, \text{ or } \frac{1}{3} < x < \frac{7}{3}$$

The solution set is $(\frac{1}{3}, \frac{7}{3})$.

26. $|3k - 2| + 1 = 8$

$$|3k - 2| = 7$$

$$3k - 2 = 7 \quad \text{or} \quad 3k - 2 = -7$$

$$3k = 9 \quad \text{or} \quad 3k = -5$$

$$k = \frac{9}{3} = 3 \quad \text{or} \quad k = -\frac{5}{3}$$

The solution set is $\{-\frac{5}{3}, 3\}$.

27. $|3 - 5x| = |2x + 8|$

$$3 - 5x = 2x + 8 \quad \text{or} \quad 3 - 5x = -(2x + 8)$$

$$-7x = 5 \quad \quad \quad 3 - 5x = -2x - 8$$

$$-3x = -11$$

$$x = -\frac{5}{7} \quad \text{or} \quad x = \frac{11}{3}$$

The solution set is $\{-\frac{5}{7}, \frac{11}{3}\}$.

28. (a) $|8x - 5| < k$

If $k < 0$, then $|8x - 5|$ would be less than a negative number. Since the absolute value of an expression is always nonnegative (positive or zero), the solution set is $\emptyset$.

(b) $|8x - 5| > k$

If $k < 0$, then $|8x - 5|$ would be greater than a negative number. Since the absolute value of an expression is always nonnegative (positive or zero), the solution set is the set of all real numbers, $(-\infty, \infty)$.

(c) $|8x - 5| = k$

If $k < 0$, then $|8x - 5|$ would be equal to a negative number. Since the absolute value of an expression is always nonnegative (positive or zero), the solution set is $\emptyset$.

Cumulative Review Exercises (Chapters 1–2)

Exercises 1–6 refer to set A .

$$\text{Let } A = \left\{-8, -\frac{2}{3}, -\sqrt{6}, 0, \frac{4}{5}, 9, \sqrt{36}\right\}.$$

Simplify $\sqrt{36} = 6$.

- The elements 9 and 6 are natural numbers.
- The elements 0, 9, and 6 are whole numbers.
- The elements -8 , 0, 9, and 6 are integers.
- The elements -8 , $-\frac{2}{3}$, 0, $\frac{4}{5}$, 9, and 6 are rational numbers.
- The element $-\sqrt{6}$ is an irrational number.
- All the elements in set A are real numbers.

$$\begin{aligned} 7. \quad -\frac{4}{3} - \left(-\frac{2}{7}\right) &= -\frac{4}{3} + \frac{2}{7} \\ &= -\frac{28}{21} + \frac{6}{21} \\ &= -\frac{22}{21} \end{aligned}$$

$$\begin{aligned} 8. \quad |-4| - |2| + |-6| &= 4 - 2 + 6 \\ &= 2 + 6 \\ &= 8 \end{aligned}$$

$$9. \quad (-2)^4 + (-2)^3 = 16 + (-8) = 8$$

$$10. \quad \sqrt{25} - 5(-1)^0 = 5 - 5(1) = 5 - 5 = 0$$

$$11. \quad (-3)^5 = (-3)(-3)(-3)(-3)(-3) = -243$$

$$12. \quad \left(\frac{6}{7}\right)^3 = \frac{6}{7} \cdot \frac{6}{7} \cdot \frac{6}{7} = \frac{216}{343}$$

$$13. \quad \left(-\frac{2}{3}\right)^3 = \left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right) = -\frac{8}{27}$$

$$14. \quad -4^6 = -(4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4) = -4096$$

$$\begin{aligned} 15. \quad -\sqrt{36} &= -(6) = -6 \\ \sqrt{-36} &\text{ is not a real number.} \end{aligned}$$

$$\begin{aligned} 16. \quad \frac{4-4}{4+4} &= \frac{0}{8} = 0 \\ \frac{4+4}{4-4} &= \frac{8}{0}, \text{ which is undefined.} \end{aligned}$$

For Exercises 17–19, let $a = 2$, $b = -3$, and $c = 4$.

$$\begin{aligned} 17. \quad -3a + 2b - c &= -3(2) + 2(-3) - 4 \\ &= -6 - 6 - 4 \\ &= -16 \end{aligned}$$

$$\begin{aligned} 18. \quad -8(a^2 + b^3) &= -8[2^2 + (-3)^3] \\ &= -8[4 + (-27)] \\ &= -8(-23) \\ &= 184 \end{aligned}$$

$$\begin{aligned}
 19. \quad \frac{3a^3 - b}{4 + 3c} &= \frac{3(2)^3 - (-3)}{4 + 3(4)} \\
 &= \frac{3(8) - (-3)}{4 + 3(4)} \\
 &= \frac{24 + 3}{4 + 12} \\
 &= \frac{27}{16}
 \end{aligned}$$

$$\begin{aligned}
 20. \quad -7r + 5 - 13r + 12 \\
 &= -7r - 13r + 5 + 12 \\
 &= (-7 - 13)r + (5 + 12) \\
 &= -20r + 17
 \end{aligned}$$

$$\begin{aligned}
 21. \quad -(3k + 8) - 2(4k - 7) + 3(8k + 12) \\
 &= -3k - 8 - 8k + 14 + 24k + 36 \\
 &= -3k - 8k + 24k - 8 + 14 + 36 \\
 &= 13k + 42
 \end{aligned}$$

$$\begin{aligned}
 22. \quad (a + b) + 4 &= 4 + (a + b) \\
 \text{The order of the terms } (a + b) \text{ and } 4 \text{ have been} \\
 \text{reversed. This is an illustration of the commutative} \\
 \text{property.}
 \end{aligned}$$

$$\begin{aligned}
 23. \quad 4x + 12x &= (4 + 12)x \\
 \text{The common variable, } x, \text{ has been removed from} \\
 \text{each term. This is an illustration of the distributive} \\
 \text{property.}
 \end{aligned}$$

$$\begin{aligned}
 24. \quad -4x + 7(2x + 3) &= 7x + 36 \\
 -4x + 14x + 21 &= 7x + 36 \\
 10x + 21 &= 7x + 36 \\
 3x &= 15 \\
 x &= 5
 \end{aligned}$$

The solution set is $\{5\}$.

$$\begin{aligned}
 25. \quad -\frac{3}{5}x + \frac{2}{3}x &= 2 \\
 3(-3x) + 5(2x) &= 15(2) \quad \text{Multiply by 15.} \\
 -9x + 10x &= 30 \\
 x &= 30
 \end{aligned}$$

The solution set is $\{30\}$.

$$\begin{aligned}
 26. \quad 0.06x + 0.03(100 + x) &= 4.35 \\
 6x + 3(100 + x) &= 435 \quad \text{Multiply by 100.} \\
 6x + 300 + 3x &= 435 \\
 9x &= 135 \\
 x &= 15
 \end{aligned}$$

The solution set is $\{15\}$.

$$\begin{aligned}
 27. \quad \text{Solve } P &= a + b + c \text{ for } b. \\
 P - a - c &= b \quad \text{or} \quad b = P - a - c
 \end{aligned}$$

$$\begin{aligned}
 28. \quad 4(2x - 6) + 3(x - 2) &= 11x + 1 \\
 8x - 24 + 3x - 6 &= 11x + 1 \\
 11x - 30 &= 11x + 1 \\
 -30 &= 1 \quad \text{False}
 \end{aligned}$$

The solution set is $\emptyset$.

$$\begin{aligned}
 29. \quad \frac{2}{3}x + \frac{5}{8}x &= \frac{31}{24}x \\
 \text{Multiply by the LCD, 24.} \\
 8(2x) + 3(5x) &= 31x \\
 16x + 15x &= 31x \\
 31x &= 31x \quad \text{True}
 \end{aligned}$$

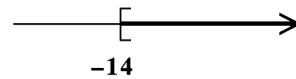
The solution set is $\{\text{all real numbers}\}$.

$$\begin{aligned}
 30. \quad 3 - 2(x + 7) &\leq -x + 3 \\
 3 - 2x - 14 &\leq -x + 3 \\
 -2x - 11 &\leq -x + 3 \\
 -x &\leq 14
 \end{aligned}$$

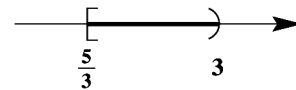
Multiply by -1 , and reverse the inequality symbol.

$$x \geq -14$$

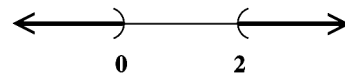
The solution set is $[-14, \infty)$.



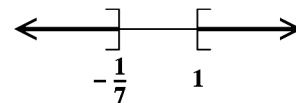
$$\begin{aligned}
 31. \quad -4 < 5 - 3x &\leq 0 \\
 -9 < -3x &\leq -5 \\
 \text{Divide by } -3, \text{ and reverse the inequality symbol.} \\
 3 > x \geq \frac{5}{3} \quad \text{or} \quad \frac{5}{3} \leq x < 3 \\
 \text{The solution set is } \left[\frac{5}{3}, 3\right).
 \end{aligned}$$



$$\begin{aligned}
 32. \quad 2x + 1 > 5 \quad \text{or} \quad 2 - x > 2 \\
 2x > 4 \quad \quad \quad -x > 0 \\
 x > 2 \quad \text{or} \quad x < 0 \\
 \text{The solution set is } (-\infty, 0) \cup (2, \infty).
 \end{aligned}$$



$$\begin{aligned}
 33. \quad |-7k + 3| &\geq 4 \\
 -7k + 3 &\geq 4 \quad \text{or} \quad -7k + 3 \leq -4 \\
 -7k &\geq 1 \quad \quad \quad -7k \leq -7 \\
 k &\leq -\frac{1}{7} \quad \text{or} \quad k \geq 1 \\
 \text{The solution set is } (-\infty, -\frac{1}{7}] \cup [1, \infty).
 \end{aligned}$$



34. Let x = the amount invested at 5%. Then
 $x + 2000$ = the amount invested at 6%.

Use $I = prt$ and create a table.

Principal	Rate (as a decimal)	Interest
x	0.05	$0.05x$
$x + 2000$	0.06	$0.06(x + 2000)$
	Total →	\$670

From the last column:

$$\begin{aligned} 0.05x + 0.06(x + 2000) &= 670 \\ 5x + 6(x + 2000) &= 67,000 \\ &\text{Multiply by 100.} \\ 5x + 6x + 12,000 &= 67,000 \\ 11x &= 55,000 \\ x &= 5000 \end{aligned}$$

She invested \$5000 at 5% and \$7000 at 6%.

35. Let x = the number of grams of food C. Then
 $2x$ is the number of grams of food A.
 There are 5 grams of food B and the total is at
 most 24 grams.

$$\begin{aligned} x + 2x + 5 &\leq 24 \\ 3x &\leq 19 \\ x &\leq \frac{19}{3} \quad \text{or} \quad 6\frac{1}{3} \end{aligned}$$

He may use at most $6\frac{1}{3}$ grams of food C.

36. Let x = the grade the student must
 make on the third test.
 To find the average of the three tests, add them
 and divide by 3. This average must be at least 80.

$$\begin{aligned} \frac{88 + 78 + x}{3} &\geq 80 \\ \frac{166 + x}{3} &\geq 80 \\ 166 + x &\geq 240 \\ x &\geq 74 \end{aligned}$$

She must score at least 74 on her third test.

37. Let x = the amount of pure alcohol
 that should be added.

Liters of Solution	Percent (as a decimal)	Liters of Pure Alcohol
x	1.00	$1.00x$
7	0.10	$0.10(7) = 0.7$
$x + 7$	0.30	$0.30(x + 7)$

From the last column:

$$\begin{aligned} 1.00x + 0.7 &= 0.30(x + 7) \\ 10x + 7 &= 3(x + 7) && \text{Multiply by 10.} \\ 10x + 7 &= 3x + 21 \\ 7x &= 14 \\ x &= 2 \end{aligned}$$

2 L of pure alcohol should be added to the
 solution.

38. Let x = the number of nickels. Then $x - 4$ = the
 number of quarters. The collection contains 29
 coins, so the number of pennies is

$$29 - x - (x - 4) = 33 - 2x.$$

	Number of Coins	Denomination	Value
Pennies	$33 - 2x$	0.01	$0.01(33 - 2x)$
Nickels	x	0.05	$0.05x$
Quarters	$x - 4$	0.25	$0.25(x - 4)$
Totals	29		\$2.69

From the last column:

$$\begin{aligned} 0.01(33 - 2x) + 0.05x + 0.25(x - 4) &= 2.69 \\ 1(33 - 2x) + 5x + 25(x - 4) &= 269 && \text{Multiply by 100.} \\ 33 - 2x + 5x + 25x - 100 &= 269 \\ 28x - 67 &= 269 \\ 28x &= 336 \\ x &= 12 \end{aligned}$$

There are $33 - 2(12) = 9$ pennies, 12 nickels, and
 $12 - 4 = 8$ quarters.

39. Clark's rule:

$$\frac{\text{Weight of child in pounds}}{150} \times \text{adult dose} = \text{child's dose}$$

If the child weighs 55 lb and the adult dosage is
 120 mg, then

$$\frac{55}{150} \times 120 = 44.$$

The child's dosage is 44 mg.

40. (a) 1990: 1611

2003: 1456

$$1611 - 1456 = 155$$

The number decreased by 155 newspapers.

(b) $\frac{155}{1611} \approx 0.096$, or 9.6%.

The number decreased by approximately 9.6%.