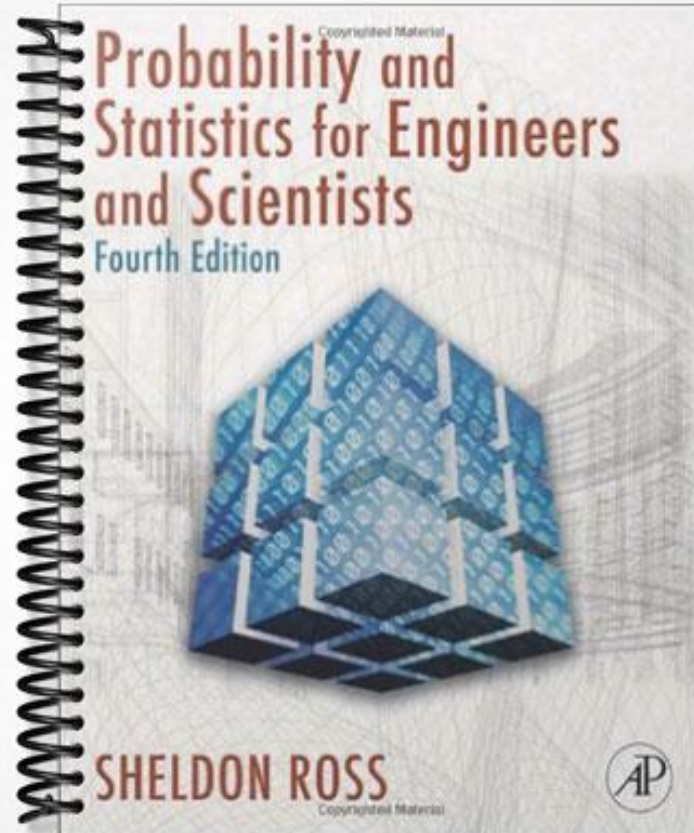


SOLUTIONS MANUAL



Instructor's Manual for

**INTRODUCTION TO
PROBABILITY AND STATISTICS
FOR ENGINEERS AND SCIENTISTS**

Fourth Edition

Sheldon M. Ross

**Department of Industrial Engineering
and Operations Research**

University of California, Berkeley



ELSEVIER

AMSTERDAM • BOSTON • HEIDELBERG • LONDON
NEW YORK • OXFORD • PARIS • SAN DIEGO
SAN FRANCISCO • SINGAPORE • SYDNEY • TOKYO

Academic Press is an imprint of Elsevier



Elsevier Academic Press
30 Corporate Drive, Suite 400, Burlington, MA 01803, USA
525 B Street, Suite 1900, San Diego, California 92101-4495, USA
84 Theobald's Road, London WC1X 8RR, UK

Copyright © 2009, Elsevier Inc. All rights reserved.

No part of this publication may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopy, recording, or any information storage and retrieval system, without permission in writing from the publisher.

Permissions may be sought directly from Elsevier's Science & Technology Rights Department in Oxford, UK: phone: (+44) 1865 843830, fax: (+44) 1865 853333, E-mail: permissions@elsevier.co.uk. You may also complete your request on-line via the Elsevier homepage (<http://elsevier.com>), by selecting “Customer Support” and then “Obtaining Permissions.”

Library of Congress Cataloging-in-Publication Data

Application submitted

ISBN 13: 978-0-12-370496-2

For all information on all Elsevier Academic Press publications
visit our Web site at www.elsevierdirect.com

09 10 9 8 7 6 5 4 3 2 1

Working together to grow
libraries in developing countries

www.elsevier.com | www.bookaid.org | www.sabre.org

ELSEVIER

BOOK AID
International

Sabre Foundation

Table of Contents

Chapter 1	1
Chapter 2	2
Chapter 3	4
Chapter 4	10
Chapter 5	17
Chapter 6	22
Chapter 7	25
Chapter 8	31
Chapter 9	37
Chapter 10	41
Chapter 11	44
Chapter 12	45
Chapter 13	47
Chapter 14	49
Chapter 15	52

Chapter 1

1. Method (c) is probably best, with (e) being the second best.
2. In 1936 only upper middle class and rich people had telephones. Almost voters have telephones today.
3. No, these people must have been prominent to have their obituaries in the Times; as a result they were probably less likely to have died young than a randomly chosen person.
4. Locations (i) and (ii) are clearly inappropriate; location (iii) is probably best.
5. No, unless it believed that whether a person returned the survey was independent of that person's salary; probably a dubious assumption.
6. No, not without additional information as to the percentages of pedestrians that wear light and that wear dark clothing at night.
7. He is assuming that the death rates observed in the parishes mirror that of the entire country.
8. $12,246/.02 = 612,300$
9. Use them to estimate, for each present age x , the quantity $A(x)$, equal to the average additional lifetime of an individual presently aged x . Use this to calculate the average amount that will be paid out in annuities to such a person and then charge that person $1 + a$ times that latter amount as a premium for the annuity. This will yield an average profit rate of a per annuity.
10. 64 percent, 10 percent, and 48 percent.

Chapter 2

2. $360/r$ degrees.
6. (d) 3.18
(e) 3
(f) 2
(g) $\sqrt{5.39}$
7. (c) 119.14
(d) 44.5
(e) 144.785
8. 278.5
10. It implies nothing about the median salaries but it does imply that the average of the salaries at company A is greater than the average of the salaries at company B.
11. (a) 40.904
(d) 8, 48, 64
12. (a) 15.808
(b) 4.395
14. Since $\sum x_i = n\bar{x}$ and $(n-1)s^2 = \sum x_i^2 - n\bar{x}^2$, we see that if x and y are the unknown values, then $x + y = 213$ and

$$x^2 + y^2 = 5(104)^2 + 16 - 102^2 - 100^2 - 105^2 = 22,667$$

Therefore,

$$x^2 + (213 - x)^2 = 22,667$$

Solve this equation for x and then let $y = 213 - x$.

15. No, since the average value for the whole country is a weighted average where the average wage per state should be weighted by the proportion of all workers who reside in that state.
21. (a) Hopkins, MIT, Stanford, and U of Washington
(b) 221.6 million
(c) 8264.4 million².
25. (a) .3496
(b) .35
(c) .1175

- (d) no
 - (e) $3700/55 = 67.3$ percent
26. (b) 3.72067
(c) .14567
28. Not if both sexes are represented. The weights of the women should be approximately normal as should be the weights of the men, but combined data is probably bimodal.
31. No, the association of good posture and back pain incidence does not by itself imply that good posture causes back pain. Indeed, although it does not establish the reverse (that back pain results in good posture) this seems a more likely possibility.
32. One possibility is that new immigrants are attracted to higher paying states because of the higher pay.
34. If $y_i = a + bx_i$ then $y_i - \bar{y} = b(x_i - \bar{x})$, implying that

$$\frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}} = \frac{b}{\sqrt{b^2}} = \frac{b}{|b|}$$

35. If $u_i = a + bx_i$, $v_i = c + dy_i$ then

$$\sum (u_i - \bar{u})(v_i - \bar{v}) = bd \sum (x_i - \bar{x})(y_i - \bar{y})$$

and

$$\sum (u_i - \bar{u})^2 = b^2 \sum (x_i - \bar{x})^2, \quad \sum (v_i - \bar{v})^2 = d^2 \sum (y_i - \bar{y})^2$$

Hence,

$$r_{u,v} = \frac{bd}{|bd|} r_{x,y}$$

36. More likely, taller children tend to be older and that is why they had higher reading scores.
37. Because there is a positive correlation does not mean that one is a cause of the other. There are many other potential factors. For instance, mothers that breast feed might be more likely to be members of higher income families than mothers that do not breast feed.

Chapter 3

1. $S = \{rr, rb, rg, br, bb, bg, gr, gb, gg\}$ when done with replacement and $S = \{rb, rg, br, bg, gr, gb\}$ when done without replacement, where rb means, for instance, that the first marble is red and the second green.
2. $S = \{hhh, hht, hth, htt, thh, tht, tth, ttt\}$. The event $\{hhh, hht, hth, thh\}$ corresponds to more heads than tails.
3. (a) $\{7\}$, (b) $\{1, 3, 4, 5, 7\}$, (c) $\{3, 5, 7\}$, (d) $\{1, 3, 4, 5\}$, (e) $\{4, 6\}$, (f) $\{1, 4\}$
4. $EF = \{(1, 2), (1, 4), (1, 6)\}$; $E \cup F = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)\}$, or any of the 15 possibilities where the first die is not 1 and the second die is odd when the first is even and even when the first is odd.; $FG = \{(1, 4)\}$; $EF^c = \{\text{any of the 15 possible outcomes where the first die is not 1 and the two dice are not either both even or both odd}\}$; $EFG = FG$.
5. (a) $2^4 = 16$
 (b) $\{(1, 1, 0, 0), (1, 1, 0, 1), (1, 1, 1, 0), (1, 1, 1, 1), (0, 0, 1, 1), (0, 1, 1, 1), (1, 0, 1, 1)\}$
 (c) $2^2 = 4$
6. (a) EF^cG^c (b) EF^cG (c) $E \cup F \cup G$ (d) $EF \cup EG \cup FG$
 (e) EFG (f) $E^cF^cG^c$ (g) $E^cF^c \cup E^cG^c \cup F^cG^c$
 (h) $(EFG)^c$ (i) $EFG^c \cup EF^cG \cup E^cFG$ (j) S
7. (a) S (b) 0 (c) E (d) EF (e) $F \cup EG$
9. $1 = EF^cG^c$ $2 = EFG^c$ $3 = E^cFG^c$ $4 = EFG$ $5 = E^cFG$ $6 = E^cF^cG$
 $7 = EF^cG$
10. Since $E \subset F$ it follows that $F = E \cup E^cF$ and since E and E^cF are mutually exclusive we have that

$$P(F) = P(E) + P(E^cF) \geq P(E)$$

11. Write $\cup E_i$ as the union of mutually exclusive events as follows:

$$\cup E_i = E_1 \cup E_1^c E_2 \cup E_1^c E_2^c E_3 \cup \dots \cup E_1^c E_{n-1}^c E_n$$

Now apply Axiom 3 and the results of Problem 10.

12. $1 \geq P(E \cup F) = P(E) + P(F) - P(EF)$

13. (i) Write $E = EF \cup EF^c$ and apply Axiom 3.

(ii) $P(E^c F^c) = P(E^c) - P(E^c F)$ from part (i)
 $= 1 - P(E) - [P(F) - P(EF)]$

14. $P(EF^c \cup E^c F) = P(EF^c) + P(E^c F)$
 $= P(E) - P(EF) + P(F) - P(EF)$ from Problem 13(i)

15. 84, 84, 21, 21, 120

16. To select r items from a set of n is equivalent to choosing the set of $n - r$ unselected elements.

17.
$$\binom{n-1}{r-1} + \binom{n-1}{r} = \frac{(n-1)!}{(n-r)!(r-1)!} + \frac{(n-1)!}{(n-1-r)!r!}$$

$$= \frac{n!}{(n-r)!r!} \left\{ \frac{r}{n} + \frac{n-r}{n} \right\} = \binom{n}{r}$$

18. (a) 1/3 (b) 1/3 (c) 1/15

19. Because the 253 events that persons i and j have the same birthday are not mutually exclusive.

20. $P(\text{smaller of } (A, B) < C) = P(\text{smallest of the 3 is either } A \text{ or } B) = 2/3$

21. (a)
$$P(A \cup B) = P(A \cup B|A)P(A) + P(A \cup B|A^c)P(A^c)$$

$$= 1 \cdot P(A) + P(B|A^c)P(A^c)$$

$$= .6 + .1(.4) = .64$$

(b) Assuming that the events A and B are independent, $P(B|A^c) = P(B)$ and

$$P(AB) = P(A)P(B) = .06$$

22. Chebyshev's inequality yields that at least $1 - 1/4$ of the accountants have salaries between \$90,000 and \$170,000. Consequently, the probability that a randomly chosen accountant will have a salary in this range is at least $3/4$. Because a salary above \$160,000 would exceed the sample mean by 1.5 sample standard deviation, it follows from the one-sided Chebyshev inequality that at most $\frac{1}{1+9/4} = 4/13$ of accountants exceed this salary. Hence, the probability that a randomly chosen accountant will have a salary that exceeds this amount is at most $4/13$.

$$\begin{aligned}
 23. \quad P(RR|\text{red side up}) &= \frac{P(RR, \text{red side up})}{P(\text{red side up})} \\
 &= \frac{P(RR)P(\text{red side up}|RR)}{P(\text{red side up})} \\
 &= \frac{(1/3)(1)}{1/2} = 2/3
 \end{aligned}$$

24. $1/2$

$$\begin{aligned}
 25. \quad P(F|CS) &= \frac{P(FCS)}{P(CS)} \\
 &= \frac{.02}{.05} = 2/5 \\
 P(CS|F) &= \frac{P(FCS)}{P(F)} \\
 &= \frac{.02}{.52} = 1/26
 \end{aligned}$$

26. (a) $\frac{248}{500}$

(b) $\frac{54/500}{252/500} = \frac{54}{252}$

(c) $\frac{36/500}{248/500} = \frac{36}{248}$

27. Let D_i be the event that ratio i is defective.

$$\begin{aligned}
 P(D_2|D_1) &= \frac{P(D_1 D_2)}{P(D_1)} \\
 &= \frac{P(D_1 D_2|A)P(A) + P(D_1 D_2|B)P(B)}{P(D_1|A)P(A) + P(D_1|B)P(B)} \\
 &= \frac{.05^2(1/2) + .01^2(1/2)}{.05(1/2) + .01(1/2)} = 13/300
 \end{aligned}$$

28. (a) $\frac{6 \cdot 5 \cdot 4}{6^3} = 5/9$

(b) $1/6$ because all orderings are equally likely.

(c) $(5/9)(1/6) = 5/54$

(d) $6^3 = 216$

(e) $\frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20$

(f) $20/216 = 5/54$

$$29. \quad P(A) = P(A|W)P(W) + P(A|W^c)P(W^c) = (.85)(.9) + (.2)(.1) = .785$$

$$P(W^c|D) = \frac{P(W^cD)}{P(D)} = \frac{(.8)(.1)}{.215} = 16/43$$

30. Let N_i be the event that i balls are colored red.

$$\begin{aligned} P(N_2|R_1R_2) &= \frac{P(N_2R_1R_2)}{P(R_1R_2)} \\ &= \frac{P(R_1R_2|N_2)P(N_2)}{P(R_1R_2|N_0)P(N_0) + P(R_1R_2|N_1)P(N_1) + P(R_1R_2|N_2)P(N_2)} \\ &= \frac{1(1/4)}{0 + (1/4)(1/2) + 1(1/4)} = 2/3 \\ P(R_3|R_1R_2) &= \frac{P(R_1R_2R_3)}{P(R_1R_2)} \\ &= \frac{0 + (1/8)(1/2) + 1(1/4)}{3/8} = 5/6 \end{aligned}$$

$$31. \quad P(D|VR) = \frac{P(DVR)}{P(VR)} = \frac{50/1000}{590/1000} = 5/59$$

32. (a) $1/3$ (b) $1/2$

33. $P\{S \text{ in second} | S \text{ in first drawer}\} = P\{A\}/P\{S \text{ in first}\}$
 $P\{S \text{ in first}\} = P\{S \text{ in first} | A\}1/2 + P\{S \text{ in first} | B\}1/2 = 1/2 + 1/2 \times 1/2 = 3/4$
 Thus probability is $1/2 \div 3/4 = 2/3$.

$$\begin{aligned} 34. \quad P(C|E) &= \frac{P(E|C)P(C)}{P(E|C)P(C) + P(E|C^c)P(C^c)} \\ &= \frac{(.268)(.7)}{(.268)(.7) + (.145)(.3)} = .8118 \\ P(C|E^c) &= \frac{P(E^c|C)P(C)}{P(E^c|C)P(C) + P(E^c|C^c)P(C^c)} \\ &= \frac{(.732)(.7)}{(.732)(.7) + (.865)(.3)} = .6638 \end{aligned}$$

$$\begin{aligned} 35. \quad (a) \quad P\{\text{good} | O\} &= P\{\text{good}, O\} / P\{O\} \\ &= .2P\{O|\text{good}\} / [P\{O|\text{good}\}.2 + P\{O|\text{average}\}.5 + P\{O|\text{bad}\}.3] \\ &= .2 \times .95 / [.95 \times .2 + .85 \times .5 + .7 \times .3] = 190/825 \end{aligned}$$

$$36. \quad (a) \quad P\{\text{sum is } 7 | \text{first is } 4\} = P\{(4, 3)\} / P\{\text{first is } 4\} = \frac{1/36}{1/6} = 1/6 = P\{\text{sum is } 7\}.$$

(b) Same argument as in (a).

37. (a) $p_5[1 - (1 - p_1p_2)(1 - p_3p_4)]$
 (b) Conditioning on whether or not circuit 3 closes yields the answer

$$p_3[(1 - (1 - p_1)(1 - p_2))[1 - (1 - p_4)(1 - p_5)] + (1 - p_3)[1 - (1 - p_1p_4)(1 - p_2p_5)]$$
38. $1 - P(\text{at most 1 works}) = 1 - Q_1Q_2Q_3Q_4 - P_1Q_2Q_3Q_4 - Q_1P_2Q_3Q_4 - Q_1Q_2P_3Q_4 - Q_1Q_2Q_3P_4$; where $Q_1 = 1 - P_1$.
39. (a) $1/8 + 1/8 = 1/4$
 (b) $P(F \cup L) = P(F) + P(L) - P(FL) = 1/4 + 1/4 - 2/32 = 7/16$
 (c) $6/32$, since there are 6 outcomes that give the desired result.

40. Let N_i be the event that outcome i never occurs. Then

$$P(N_1 \cup N_2) = .5^n + .8^n - .3^n$$

Hence, the desired answer is $1 - .5^n + .8^n - .3^n$

41. Let W_1 be the event that component 1 works. Then,

$$P(W_1|F) = \frac{P(W_1F)}{P(F)} = \frac{P(F|W_1)(1/2)}{1 - (1/2)^n} = \frac{1/2}{1 - (1/2)^n}$$

42. 1: (a) $1/2 \times 3/4 \times 1/2 \times 3/4 \times 1/2 = 9/128$
 (b) $1/2 \times 3/4 \times 1/2 \times 3/4 \times 1/2 = 9/128$
 (c) $18/128$
 (d) $1 - P(\text{resembles first or second}) = 1 - [9/128 + 9/128 - P(\text{resembles both})] = 110/128$
- 2: (a) $1/2 \times 1/2 \times 1/2 \times 1/2 \times 1/2 = 1/32$
 (b) $1/32$ (c) $1/16$ (d) $1 - 2/32 = 15/16$

43. Prisoner A's probability of being executed remains equal to $1/3$ provided the jailer is equally likely to answer either B or C when A is the one to be executed. To see this suppose that the jailer tells A that B is to be set free. Then

$$\begin{aligned} P\{A \text{ to be executed} | \text{jailer says } B\} &= P\{A \text{ executed}, B\}/P\{B\} \\ &= \frac{P\{B|A \text{ executed}\}1/3}{P\{B|A \text{ exec.}\}1/3 + P\{B|C \text{ exec.}\}1/3} \\ &= 1/6 + (1/6 + 1/3) = 1/3 \end{aligned}$$

44. Since brown is dominant over blue the fact that you have blue eyes means that both your parents have one brown and one blue gene. Thus the desired probability is $1/4$.
45. $1 - (1 - p_i - p_j)^k$

46. Let 1 be the card of lowest value, 2 be the card of next higher value, and 3 be the card of highest value.

(a) $1/3$, since the first card is equally likely to be any of the 3 cards.

(b) You will accept the highest value card if the cards appear in any of the orderings;

1, 3, 2 or 2, 3, 1 or 2, 1, 3

Thus, with probability $3/6$ you will accept the highest valued card.

47. $.2 + .3 = .5$, $.2 + .3 - (.2)(.3) = .44$, $.2(.3)(.4) = .024$, 0

48. Let C be the event that the woman has breast cancer. Then

$$\begin{aligned} P(C|pos) &= \frac{P(C, pos)}{P(pos)} \\ &= \frac{P(pos|C)P(C)}{P(pos|C)P(C) + P(pos|C^c)P(C^c)} \\ &= \frac{.9(.02)}{.9(.02) + .1(.98)} \\ &= \frac{18}{116} \end{aligned}$$